Introduction to MATLAB

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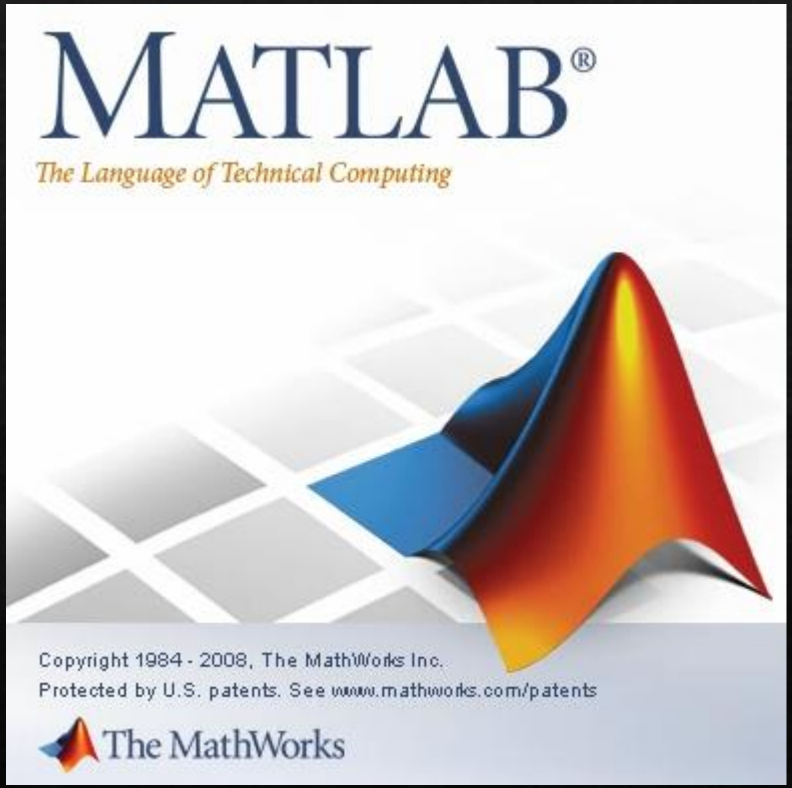
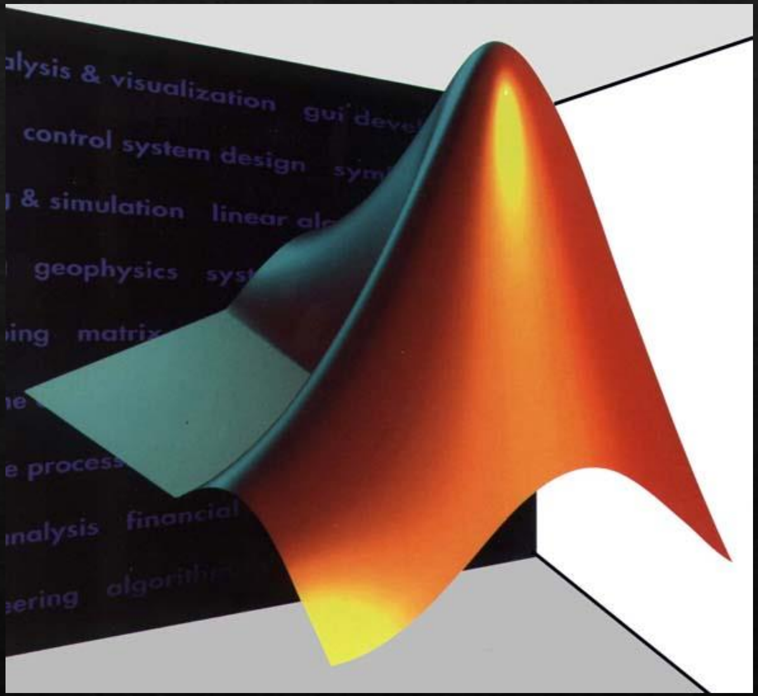
Outline

- Overview
- Matrix and array
- Language fundamentals
- Programming
- Plotting and graphics
- Mathematics
- Exercises



Overview

- A high-level language and interactive environment for numerical computation, visualization, and programming.
- Analyze data.
- Develop algorithms.
- Create models and applications.
- More than a million engineers and scientists in industry and academia use Matlab.
- The language of technical computing.
- Commercial !

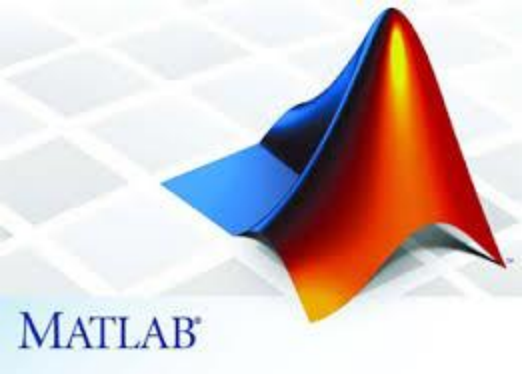


Key Features

- Interactive environment for iterative exploration, design, and problem solving.
- Mathematical functions for linear algebra, statistics, Fourier analysis, filtering, optimization, numerical integration, and solving ordinary differential equations.
- Built-in graphics for visualizing data and tools for creating custom plots.
- Development tools for improving code quality and maintainability and maximizing performance.
- Tools for building applications with custom graphical interfaces.
- Functions for integrating MATLAB based algorithms with external applications and languages such as C, Java and Microsoft Excel.

Matlab vs. C/C++/Fortran

- Matlab **compared to C/C++ or Fortran**
  - a higher level of programming language
  - similar grammar to C/C++ (but more simplified)
  - more tools and built-in math functions
  - reach a solution for a math or numerical problem faster
  - better graphical interface
  - slower computation speed for most numerical problems, not suitable for massive computation.
  - good for post analysis of data and visualization.



Matlab vs. Mathematica

- **Matlab compared to Mathematica**
  - both have very good built-in math functions
  - both have good graphical interface
  - better at numerical treatments
  - worse at analytical treatments

    (needs additional Symbolic Math toolbox)
  - more popular in engineering
  - less popular in science

Run MATLAB on HPC clusters

- Log in and set up environments:

ssh –X username@mike.hpc.lsu.edu

softenv –k matlab

vi .soft

Add one line in .soft: +matlab-r2012b

resoft

- Interactive section:

qsub –I –X –A hpc_train_2014 –l nodes=1:ppn=16,walltime=02:00:00

matlab –nodesktop

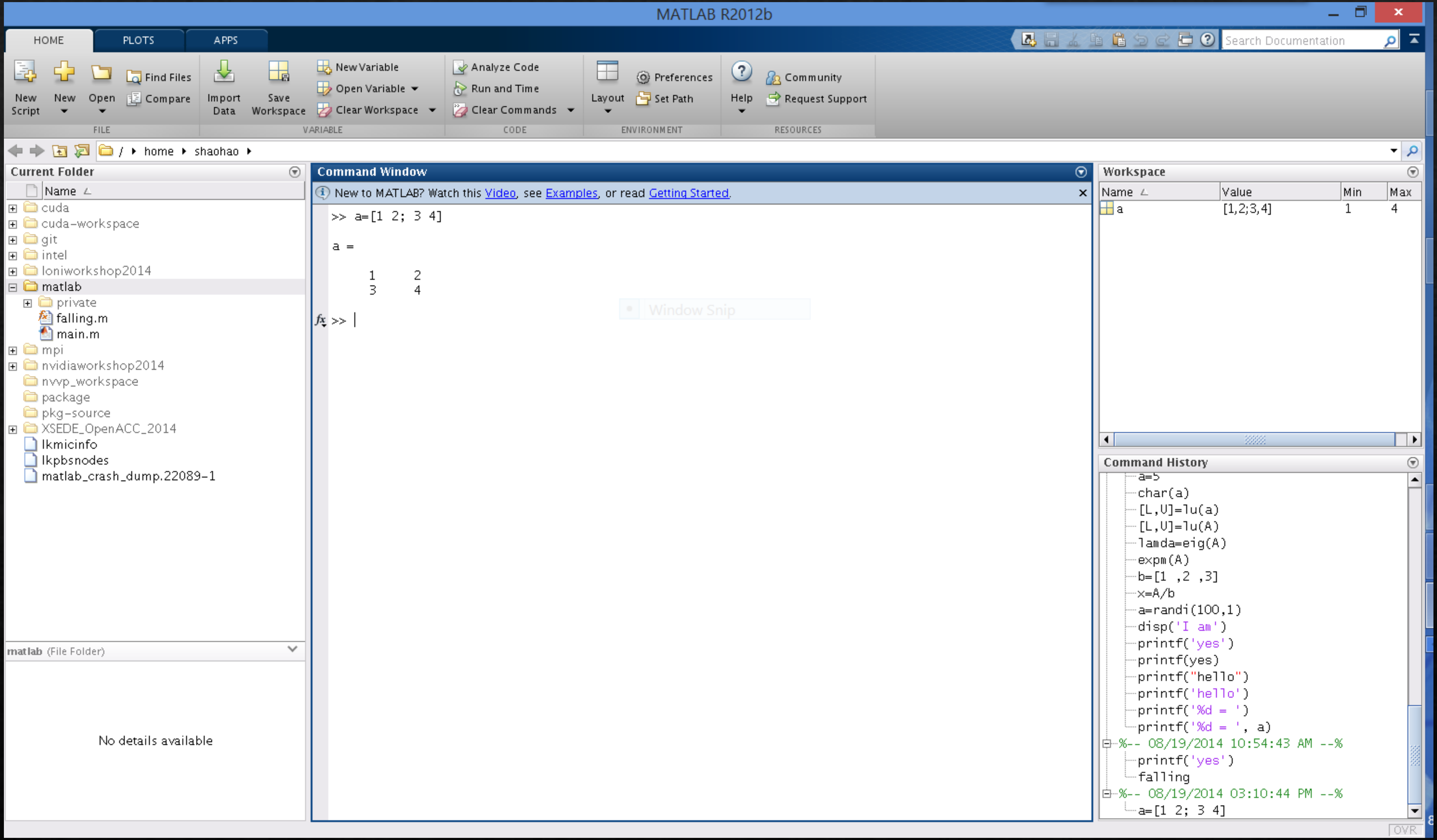
- Submit a batch job

See details at HPC@LSU website:
http://www.hpc.lsu.edu/docs/guides/software.php?software=matlab

Desktop window

- matlab &
- Some Linux commands are available.
- cd
- ls
- !cp
- !rm
- !mv
- !vim

(pre exclamation)



Matrix and arrays

- Matlab = lab of matrix
- Most ojects (e.g. data, text, color) in Matlab can be represented by a vector or a matrix.
- Convenient for linear algebra operations.

```
% use percent sign for comments,
% auto output without a semicolon
```

- **Scalar:** a = 5
- **Array (vector):** b = [1 2 3] ; % row
  c = [4; 5; 6] ; % column
- **Matrix:** A = [1 2 3; 4 5 6; 7 8 9] % row-first convention
- **Vector operations:**
  - dot(b,c) % dot product
  - cross(b,c) % cross product
  - norm(b) % norm

Matrix operations:

- A + a % a matrix plus a scalar
- A*a % a matrix multiplies a scalar
- sin(A) % sine of each element of a matrix
- exp(A) % exponential of each element of a matrix
- A’ % transpose of a matrix
- A + A % matrix addition
- A*A % matrix multiplication
- A.*A % element-by-element multiplication.
- A./A % element-by-element devision.
- A.^3 % element-by-element power
- A’ % complex conjugate transpose of a matrix
- inv(A) % inverse of a matrix
- det(A) % determination of a matrix

- **Matrices indexing**
  - A(3,2) % the element of 3rd line and 2nd colomn
  - A(:,1) % the 1st column
  - A(2,2:3) % through 2nd to 3rd elements of 2nd line
  - sum(A(2,:)) % sum all elements of the 2nd line
  - max(A(3,:)) % maximum element of the 3rd line
  - find(isprime(A)) % find prime numbers among all elements

- **Basic matrices**
  - zeros(3,3) % All zeros
  - ones(3,3) % All ones
  - rand(3,3) % Uniformly distributed random elements, between 0 and 1.
  - randn(3,3) % Normally distributed random elements
  - I=eye(3) % Unit matrix

Language fundamentals

- **Variables**
  - No declaration required (more convenient than C or Fortran)
  - n = 25 % Integer
  - a = 6.2 % Real number

- **Data type**
  - double(n) % Convert to double precision
  - single(n) % Convert to single precision
  - int8(a), int16(a), int32(a), int64(a) % Convert to 8-bit, 16-bit, 32-bit, 64-bit signed integer
  - eps % Floating-point relative accuracy: 2.2204e-16 for double

Complex number

- i % imaginary unit
- j % imaginary unit
- sqrt(-1) % imaginary unit
- x=3+4i % complex number
- x=complex(a,b) % real part is a, imaginary part is b
- complex(x) % convert to complex number
- real(x) % real part of x
- imag(x) % imaginary part of x
- angle(x) % argument of x
- abs(x) % amplitude of x
- conj(x) % conjugate of x
- isreal(x) % x is real or not

- Math expressions

- a^3+b^2-3*c+d/6+9
- abs(x)   % absolute value
- sin(x); cos(x); tan(x);   % triangle functions
- asin(x); acos(x); atan(x); atan2(y,x); % inverse triangle functions
- sqrt(x)   % square root
- exp(x)   % exponential
- log(x)   % natural logarithm
- log10(x)   % base-10 logarithm

- Long statement

- s = 1 -1/2 + 1/3 -1/4 + 1/5 - 1/6 + 1/7 ...   // combine two lines

  - 1/8 + 1/9 - 1/10 + 1/11 - 1/12;

Programming I : Control Flow

- Conditional Control — if, else

```
a = randi(100, 1);
if a < 30
    fprintf('%i is smaller than 30. \n', a);
elseif a > 80
    fprintf('%d is larger than 80. \n', a);
else
    X=[num2str(a), ' is between 30 and 80.'];
    disp(X)
end
```

- Conditional Control — switch

```
[dayNum, dayString] = weekday(date, 'long', 'en_US');
switch dayString
    case 'Monday'
        disp('Start of the work week')
    case 'Tuesday'
        disp('Day 2')
    case 'Wednesday'
        disp('Day 3')
    case 'Thursday'
        disp('Day 4')
    case 'Friday'
        disp('Last day of the work week')
    otherwise
        disp('Weekend!')
end
```

Loop Control — for

```
% sum of an array
    s=0;
    b=rand(100,1)
    for i = 1:1:100
        s=s+b(i);        % not allow +=
    end
    s
```

```
% nested loop
    m=50;
    n=100;
    for i = 1:1:m        % Stripe 1
        for j = 1:2:n    % Stripe 2
            H(i , j) = 1/(i+j);
        end
    end
```

❑ Loop Control — while, break

% find a root of the cubic polynomial x^3 - 2x – 5 using Newton's method

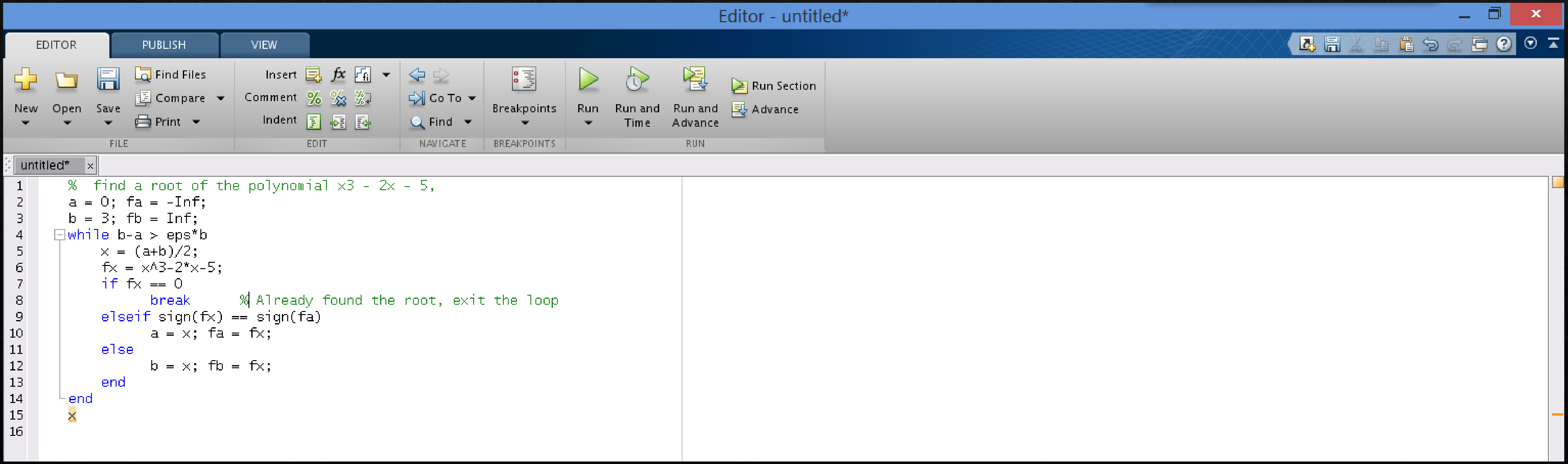
```
a = 0; fa = -Inf;
b = 3; fb = Inf;
while b-a > eps*b
   x = (a+b)/2;
   fx = x^3-2*x-5;
   if fx == 0
      break     % Already found the root, exit the loop
   elseif sign(fx) == sign(fa)
      a = x; fa = fx;
   else
      b = x; fb = fx;
   end
end
x
```

M-file and Script

- An m-file, or script file, is a simple text file where you can place MATLAB commands.
- Save your works
- Convenient for debugging
- Run directly. No explicit compilation and link.

- modify and run M-files:

!vim file_name.m
edit file_name.m
open file_name.m
run file_name.m



Matrix multiplication in Matlab

```
n = 1000;          % set matrix dimension

for i=1:1:n          % initialize data
  for j=1:1:n
        A(i,j)=i+j;
        B(i,j)=2*i-j;
  end
end

C=A*B  % matrix  multiplication & output
```

Matrix multiplication in C

```
...... /* include head files if necessary */

int main(){

  int n = 1000;        // set matrix dimension

  /* declaration and allocation */
  double *a = (double *) malloc ( n*n*sizeof(double) );
  double *b = (double *) malloc ( n*n*sizeof(double) );
  double *c = (double *) malloc ( n*n*sizeof(double) );

 /* initialize data */
  for (int i=0; i<n; i++)
  for (int j=0; j<n; j++) {
          a[i][j]=i+j;
          b[i][j]=2.*i-j;
  }

 /* matrix multiplication */
  for (int i=0; i<n; i++)          // loop over output rows, i
    for (int j=0; j<n; j++) {      // loop over output columns, j
      c[i][j] = 0;                 // initialize output result to zero
      for (k=0; k<n; k++)          // loop over inner dimension, k
        c[i][j] += a[i][k] * b[k][j];   // perform sum
  }

  ...... /* output results */

} // end program
```

- use MKL or LAPACK/BLAS to simplify the C codes

Programming II : Functions

- Anonymous Functions

  f = @(arglist) expression

- One argument

```
my_fun = @(x)  x.^2+exp(x)+5;
my_fun(5)
```

- Two arguments

```
my_fun = @(x,y)  x.^3+6*sqrt(y);
my_fun(3,4)
```

- Local Functions

    function y = my_fun(x)

```
% In falling.m . File name should be the same as function name.
function height = falling(t)
    global GRAVITY
    height = 1/2*GRAVITY*t.^2;     % The height of a freely falling object
end
```

```
% In main.m. Should be in the same path of falling.m
global GRAVITY
GRAVITY = 32;
y = falling((0:.2:5)')
```

Programming III : I/O with external files

- Input from a file

% text format

load(filename,'-ascii')

% Matlab format

load(filename,'-mat')

- Output to a file

% open a file

fileID = fopen(filename,permission)

% print data

fprintf(fileID,formatSpec,A1,...,An)

- An example

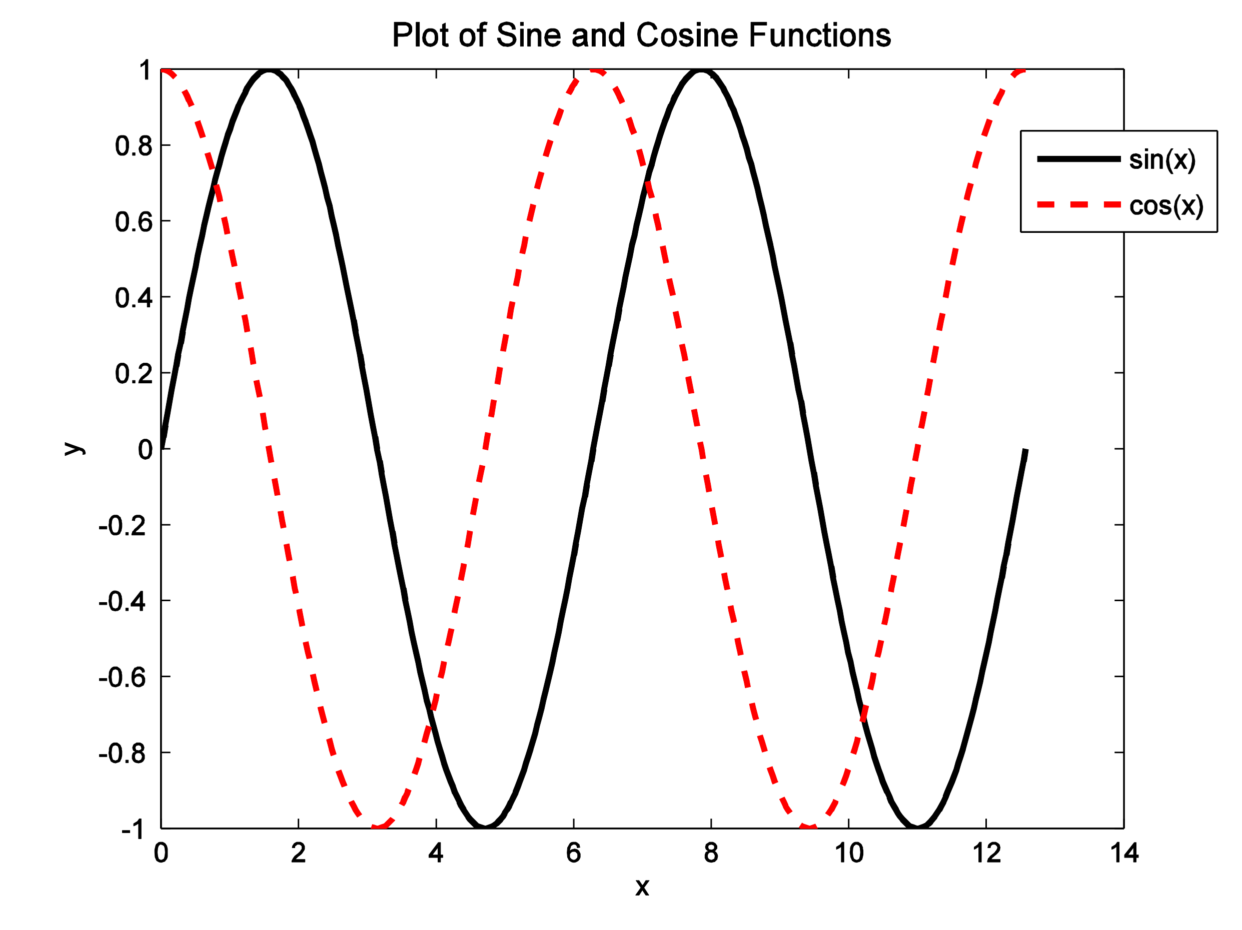
```
x = 0:.1:1;
A = [x; exp(x)];

fileID = fopen('exp.txt','w'); % open a writable file
fprintf(fileID,'%6.2f %12.8f\n',A); % print real data
fclose(fileID);       % close the file

B=load('exp.txt','-ascii') % load data from the file
```

Plotting and Graphics

- ❑ Plot figures with command lines instead of mouse! Easy for repeating.

◈ **plot**

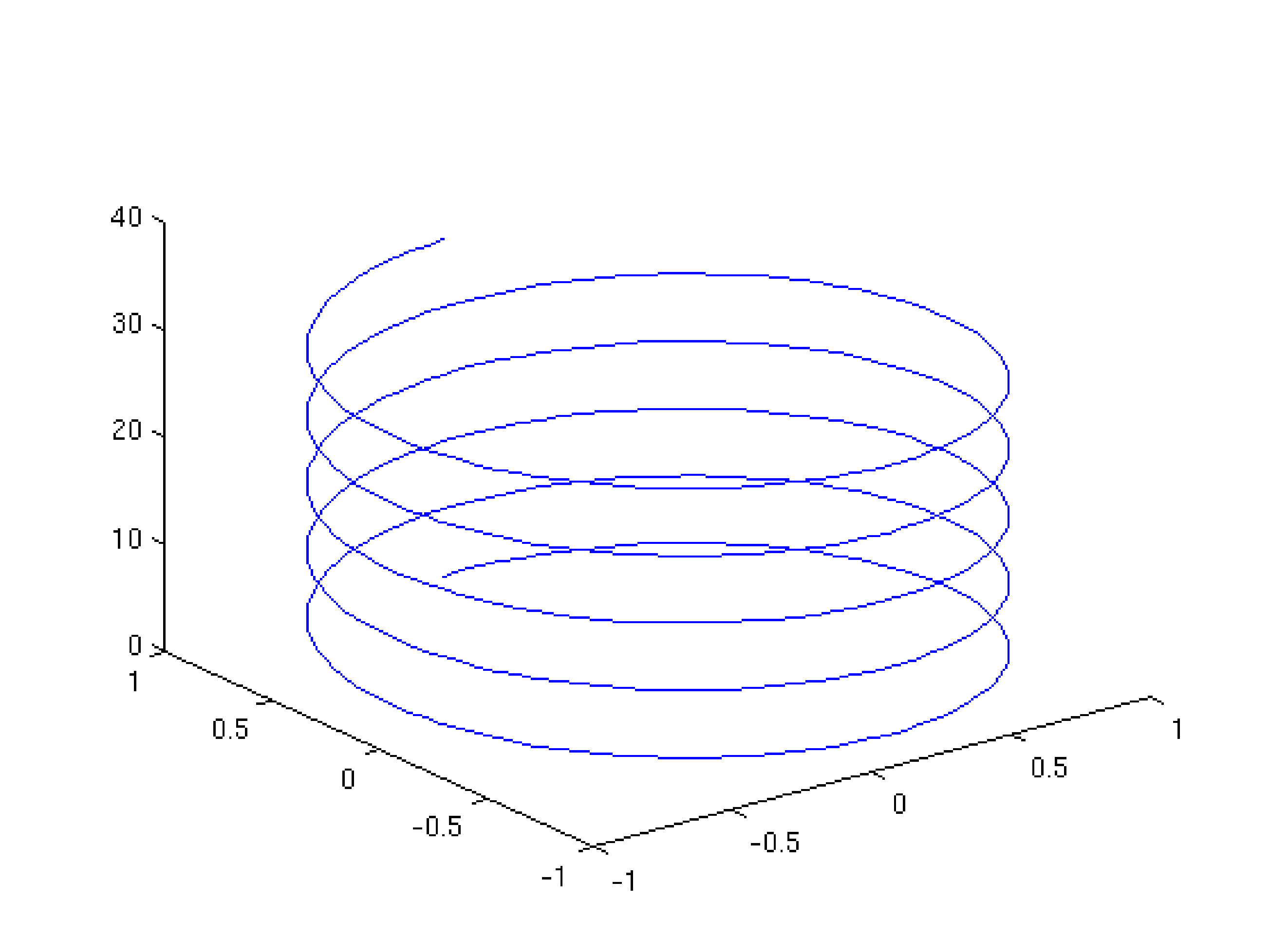
```
x = 0:pi/100:4*pi;
y = sin(x);
y2 = cos(x);
plot(x,y,'black',x,y2,'red--','linewidth',2)
xlabel('x')
ylabel('y')
axis([0 4*pi -1 1])
title('Plot of Sine and Cosine Functions','FontSize',12)
legend('sin(x)','cos(x)')
```



Plotting three-dimensional curves

- **plot3**

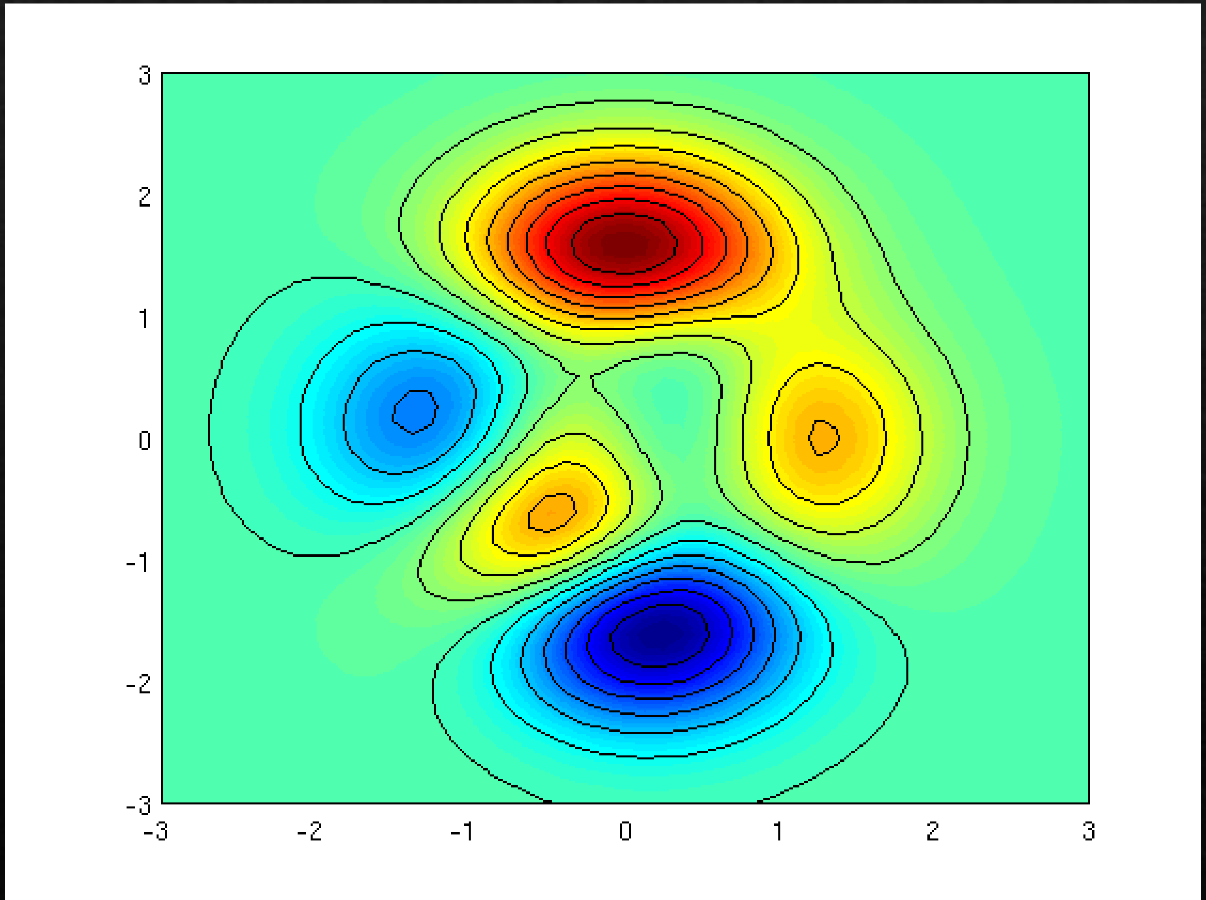
```
t = 0:pi/50:10*pi;  % z
st = sin(t);        % x
ct = cos(t);        % y

figure
plot3(st,ct,t)
```



Contour Plot

- **Contour, pcolor**

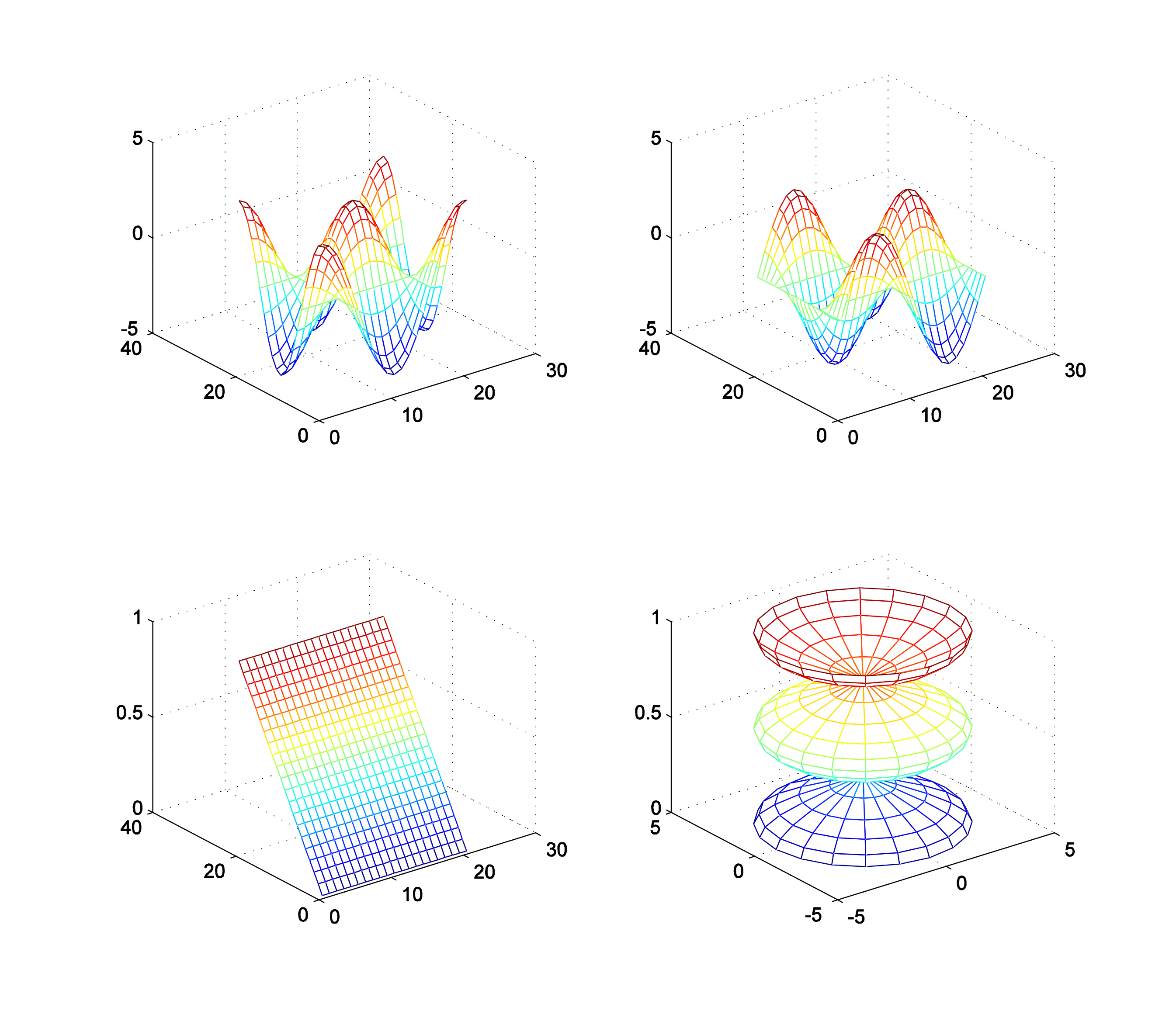
```
% Obtain data from evaluating peaks function
[x,y,z] = peaks;
% Create pseudocolor plot
pcolor(x,y,z)
% Smooth the colors
shading interp
% Hold the current graph
hold on
% Add the contour graph to the pcolor graph
contour(x,y,z,15,'k')
% Return to default
hold off
```



Subfigures and layout

◈ **Subplot, mesh**

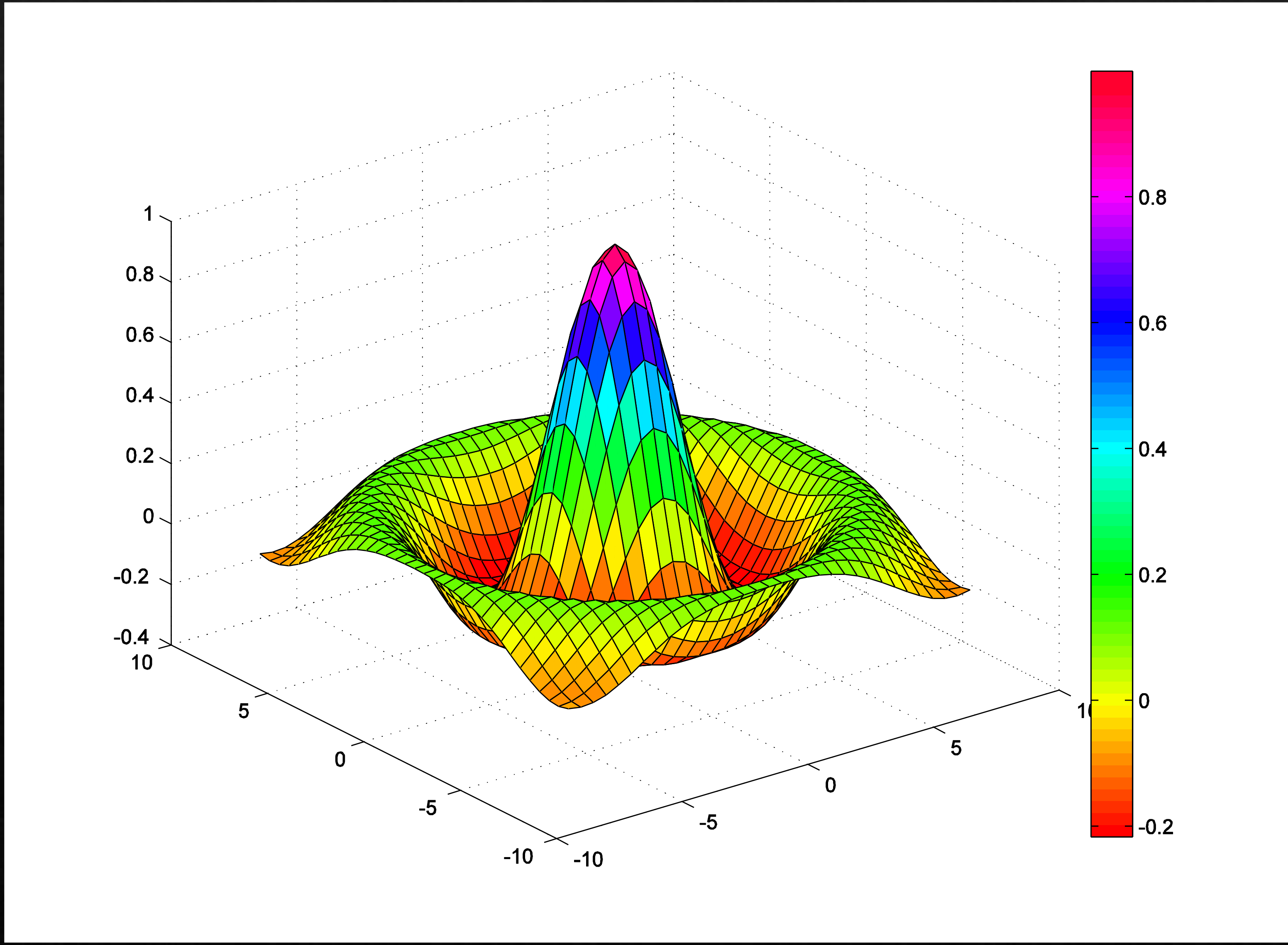
```
t = 0:pi/10:2*pi;
% cylinder with a self-defined profile
[X,Y,Z] = cylinder(4*cos(t));
subplot(2,2,1);   % left-up
mesh(X)
subplot(2,2,2);   % right-up
mesh(Y)
subplot(2,2,3);   % left-down
mesh(Z)
subplot(2,2,4);   % right-down
mesh(X,Y,Z)
```



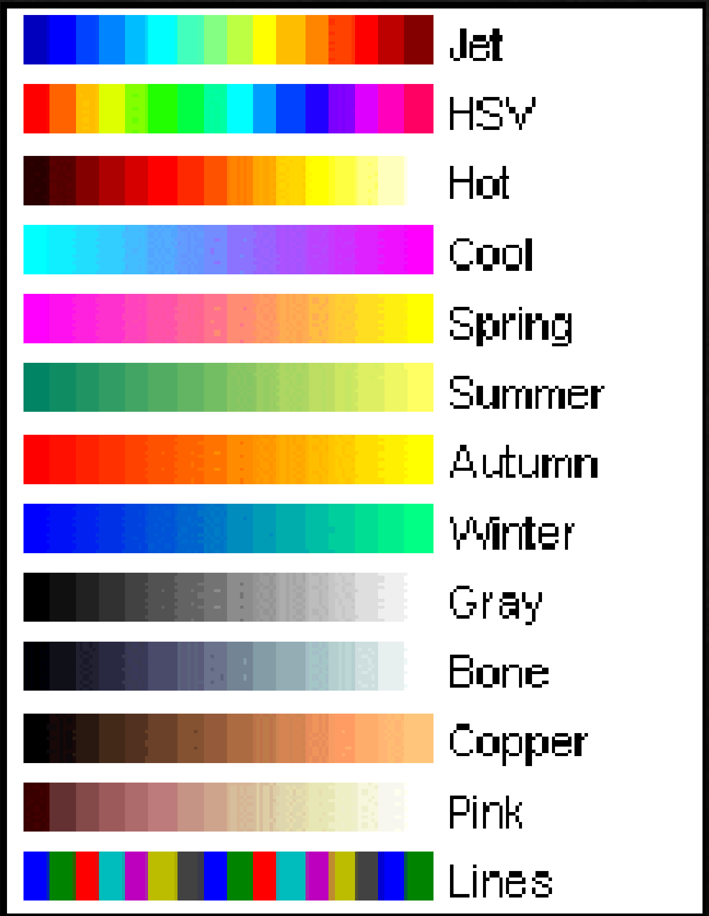
Color Surface Plot

- **surfc**

```
[X,Y] = meshgrid(-8:.5:8);
R = sqrt(X.^2 + Y.^2) + eps;
Z = sin(R)./R;   % sinc function
surfc(X,Y, Z)
shading interp
colormap hsv     % color map
colorbar    % show color scaling
view([1 1 1])  % view angle
```



- colormap



- color palette

Red	[1 0 0]
Green	[0 1 0]
Blue	[0 0 1]
Black	[0 0 0]
White	[1 1 1]

% design your own color map

- view angles

From x axis: [1 0 0]

From y axis: [0 1 0]

From z axis: [0 0 1]

From diagonal line: [1 1 1]

Animation

- **gca, getframe, movie**

```
Z = peaks;                    % create random Gaussian peaks
figure('Renderer','zbuffer');   % rendering method used for screen and printing
surf(Z);                      % plot the peaks
axis tight manual;   % set axis tight and freeze the scaling
set(gca,'NextPlot','replaceChildren'); % remove previous figures but keep the figure properties
for j = 1:20
    surf(sin(2*pi*j/20)*Z,Z)    % modify the shapes of the peaks at different time instants
    F(j) = getframe;    % capture the current figure as a movie frame
end
movie(F,10)        % play the movie ten times
```

Mathematics

- Numerical integration and differentiation
- Linear algebra equations
- Eigen problem
- Fast Fourier Transform (FFT)
- Mathematical special functions

Numerical integration

- One-dimensional integration

q = integral(fun,xmin,xmax)

% approximates the integral of function fun from xmin to xmax using global adaptive quadrature and default error tolerances.

```
fun = @(x) exp(-x.^2).*log(x).^2;   % define a function  f(x) = e^-x2 [ln(x)]^2
p = integral(fun,0,0.5)     % proper integral
q = integral(fun,0,Inf)     % improper integral

fun = @(x)log(x);           % logarithm function
format long                 % output long digits
q = integral(fun,0,1)       % integral with singularity at the lower limit
```

- Two- and three-dimensional integrations

q = integral2(fun,xmin,xmax,ymin,ymax)

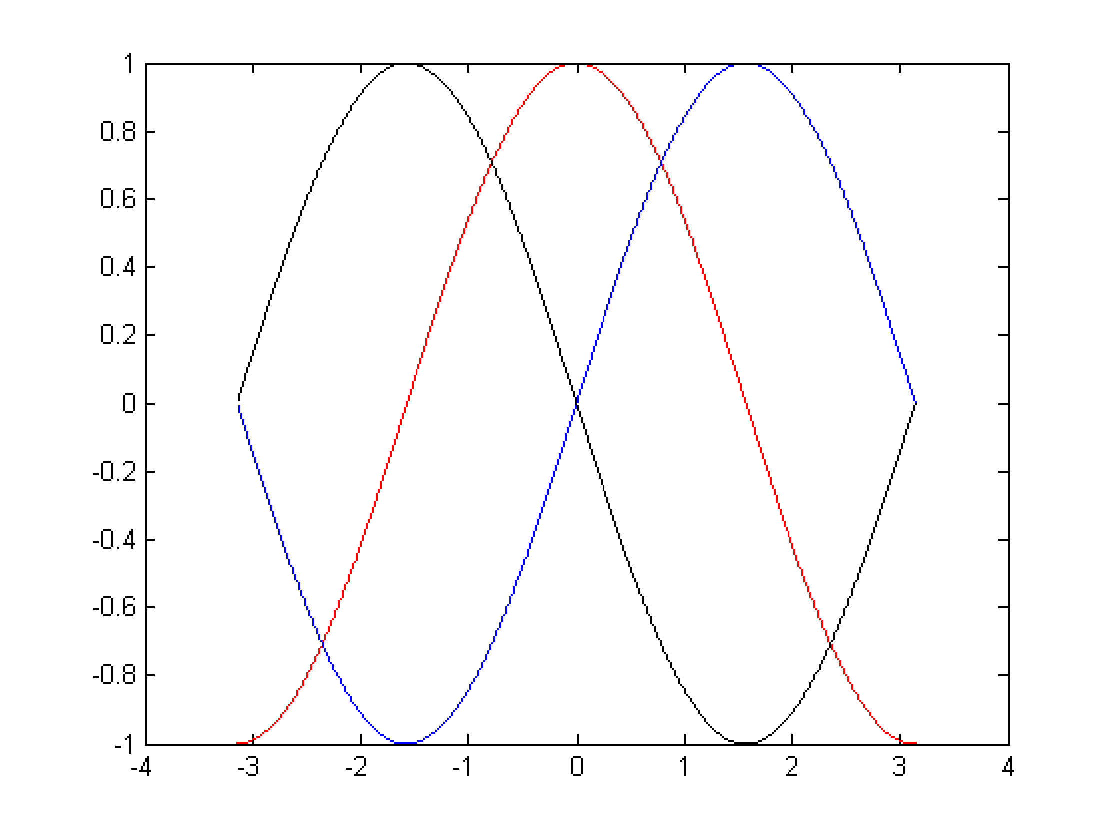
q = integral3(fun,xmin,xmax,ymin,ymax,zmin,zmax)

$$f(\theta,r) = \frac{r}{\sqrt{r\cos\theta + r\sin\theta}\,(1 + r\cos\theta + r\sin\theta)^2}$$

```
polarfun = @(theta,r) fun(r.*cos(theta),r.*sin(theta)).*r;   % define a 2D function in polar coordinates
rmax = @(theta) 1./(sin(theta) + cos(theta));     % set upper limit as a function of theta
p = integral2(polarfun,0,pi/2,0,rmax)             % 2D integral

fun = @(x,y,z) y.*sin(x)+z.*cos(x);  % define a 3D function
q = integral3(fun,0,pi,0,1,-1,1)        % 3D  integral with finite limits
```

Numerical differentiation

```
Y = diff(X)      % first devirative
Y = diff(X,n)    % the nth devirative

h = 0.001;       % step size
X = -pi:h:pi;    % domain
f = sin(X);      % range
Y = diff(f)/h;   % first derivative
Z = diff(f,2)/h^2;   % second derivative
plot(X(:,1:length(Y)),Y,'r',X,f,'b', X(:,1:length(Z)),Z,'k')
```



Solve linear algebra equations

- Solves the linear system A*x = b

```
x = A\b
x = mldivide(A,b)
x = linsolve(A,b)
x = linsolve(A,b,opts)
% Using LU factorization when A is square and QR factorization otherwise.
% The number of rows of A must equal the number of columns of B.

A = triu(rand(5,5));     % random 5*5 up-triangle matrix
b = rand(5,1);           % random column array
opts.UT = true;          % up-triangle is ture
x = linsolve(A,b,opts)
x = A\b
% The solution is straight forward, if A is triangular.
```

Factorization

- LU factorization

[L,U] = lu(A) % expresses a matrix A as the product of two essentially triangular matrices, one of them a permutation of a lower triangular matrix and the other an upper triangular matrix. The factorization is often called the LU, or sometimes the LR, factorization.

```
[L,U] = lu(A);  % obtain L and U matrices
y=L\b;
x=U\y   % equivalent to x=A\b
```

- QR factorization

[Q,R] = qr(A) % expresses a matrix A into a product A = QR of an orthogonal matrix Q and an upper triangular matrix R.

```
[Q,R] = qr(A);  % obtain Q and R matrices
y=Q'*b;
x=R\y   % equivalent to x=A\b
```

Eigen problem

- Find eigen values and eigen vectors for a matrix (A*x = lambda*x)

[V,D] = eig(A)   % returns a diagonal matrix containing the eigenvalues (D) and a matrix whose columns are the corresponding right eigenvectors (V).

- Solve eigen problem for symmetric and asymmetric matrix

```
u = triu(rand(5,5));   % create a random 5*5 up-triangle matrix
A = u+u';              % create a symmetric matrix
[P,L] = eig(A)         % The eigen values and vectors of a symmetric matrix are real.
P'*A*P                 % check: should equal to L

B = rand(5,5);         % create a random 5*5 matrix
[V,D] = eig(B)         % The eigen values and vectors could be complex.
V'*B*V                 % check: should equal to D
```

Fast Fourier Transform (FFT)

- FFT of an array

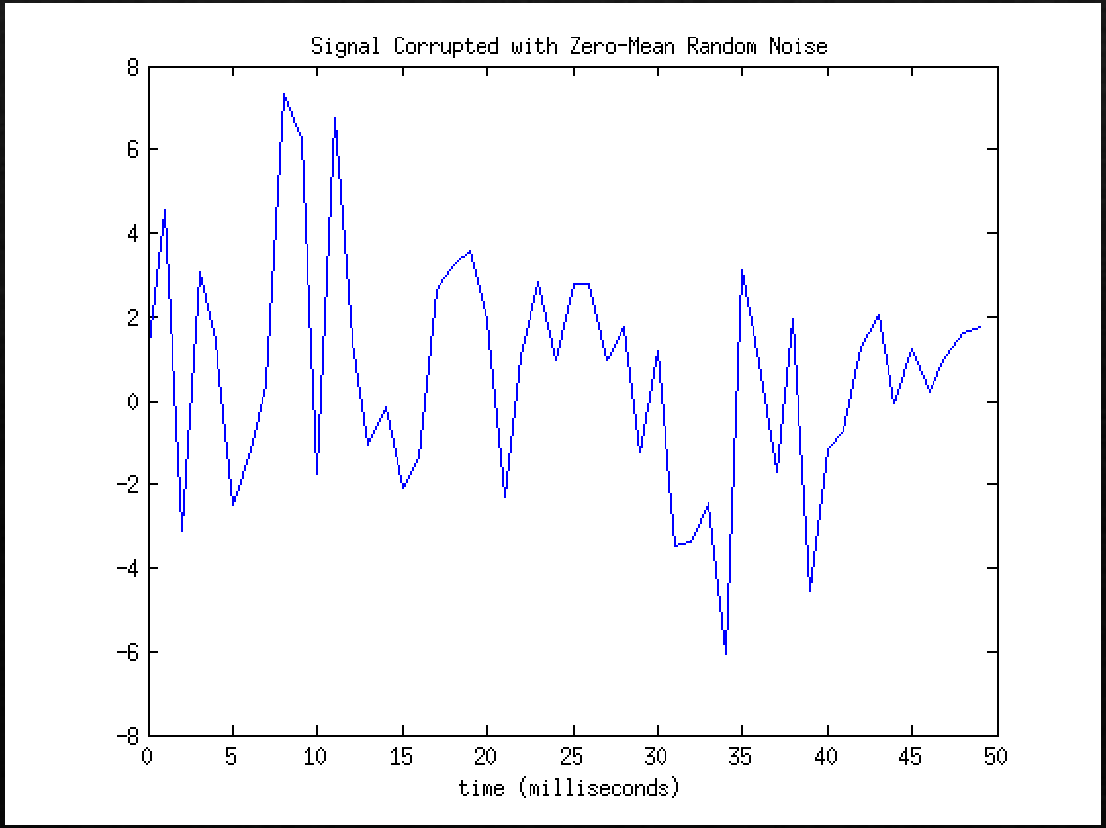
y = fft(x,n)

% FFT is an algorithm to compute the discrete Fourier transform (DFT) and its inverse.

% Fourier transform converts time (or space) to frequency (or momentum).

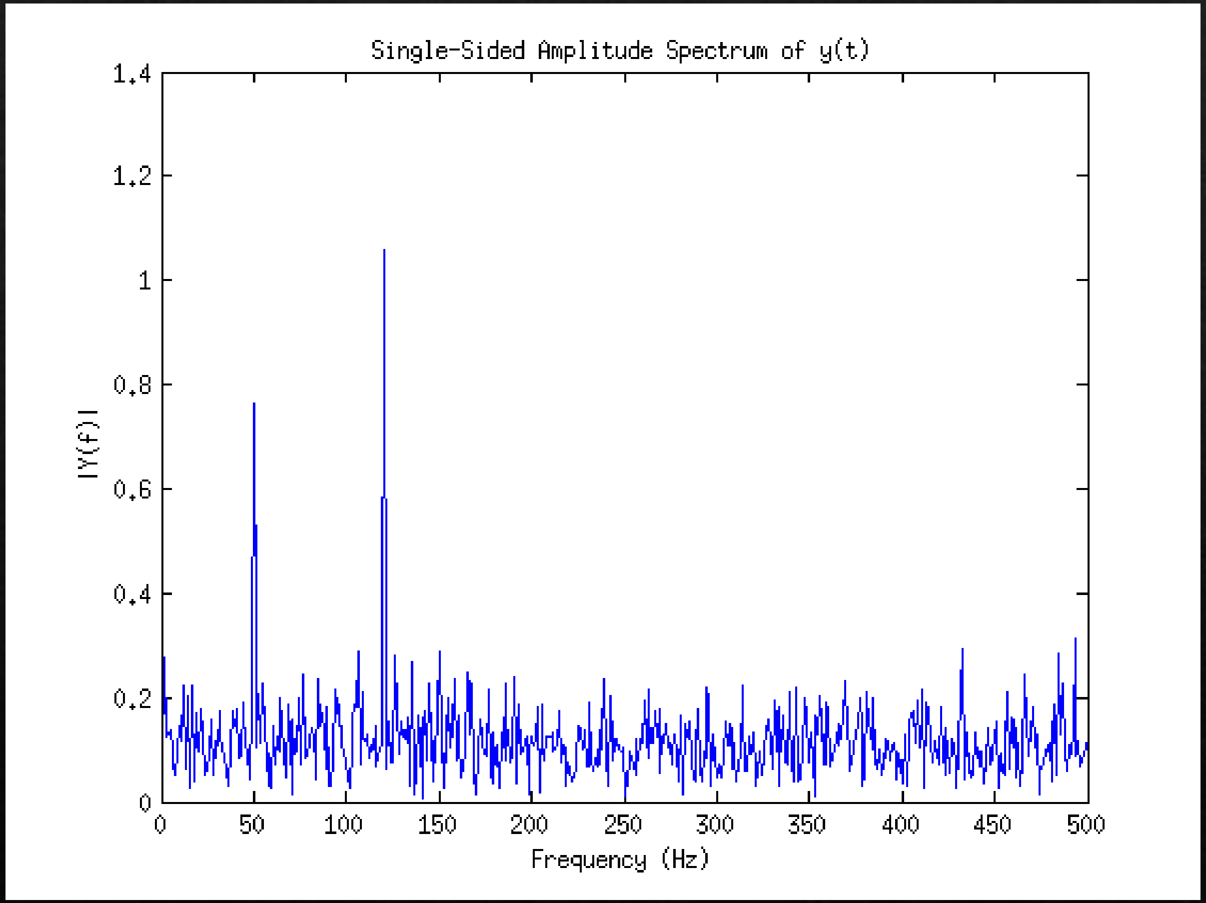
- Create a time series

```
Fs = 1000;                    % Sampling frequency
T = 1/Fs;                     % Sample time
L = 1000;                     % Length of signal
t = (0:L-1)*T;                % Time vector
% Sum of a 50 Hz sinusoid and a 120 Hz sinusoid
x = 0.7*sin(2*pi*50*t) + sin(2*pi*120*t);
y = x + 2*randn(size(t));     % Sinusoids plus noise
plot(Fs*t(1:50),y(1:50))
title('Signal Corrupted with Zero-Mean Random Noise')
xlabel('time (milliseconds)')
```



◈ Fourier analysis

```
% Next power of 2 from length of y
NFFT = 2^nextpow2(L);
Y = fft(y,NFFT)/L;
f = Fs/2*linspace(0,1,NFFT/2+1);

% Plot single-sided amplitude spectrum.
plot(f,2*abs(Y(1:NFFT/2+1)))
title('Single-Sided Amplitude Spectrum of y(t)')
xlabel('Frequency (Hz)')
ylabel('|Y(f)|')
```



Mathematical special functions

- Many special functions appear as solutions of differential equations or integrals of elementary functions.
  - airy % Airy Functions
  - besselh % Bessel function of third kind (Hankel function)
  - besseli % Modified Bessel function of first kind
  - besselj % Bessel function of first kind
  - besselk % Modified Bessel function of second kind
  - bessely % Bessel function of second kind
  - beta % Beta function
  - betainc % Incomplete beta function
  - Betaincinv % Beta inverse cumulative distribution function
  - betaln % Logarithm of beta function
  - ellipj % Jacobi elliptic functions
  - ellipke % Complete elliptic integrals of first and second kind

Mathematical special functions

erf	% Error function
erfc	% Complementary error function
erfcinv	% Inverse complementary error function
erfcx	% Scaled complementary error function
erfinv	% Inverse error function
expint	% Exponential integral
gamma	% Gamma function
gammainc	% Incomplete gamma function
gammaincinv	% Inverse incomplete gamma function
gammaln	% Logarithm of gamma function
legendre	% Associated Legendre functions
psi	% Psi (polygamma) function

Bessel Function

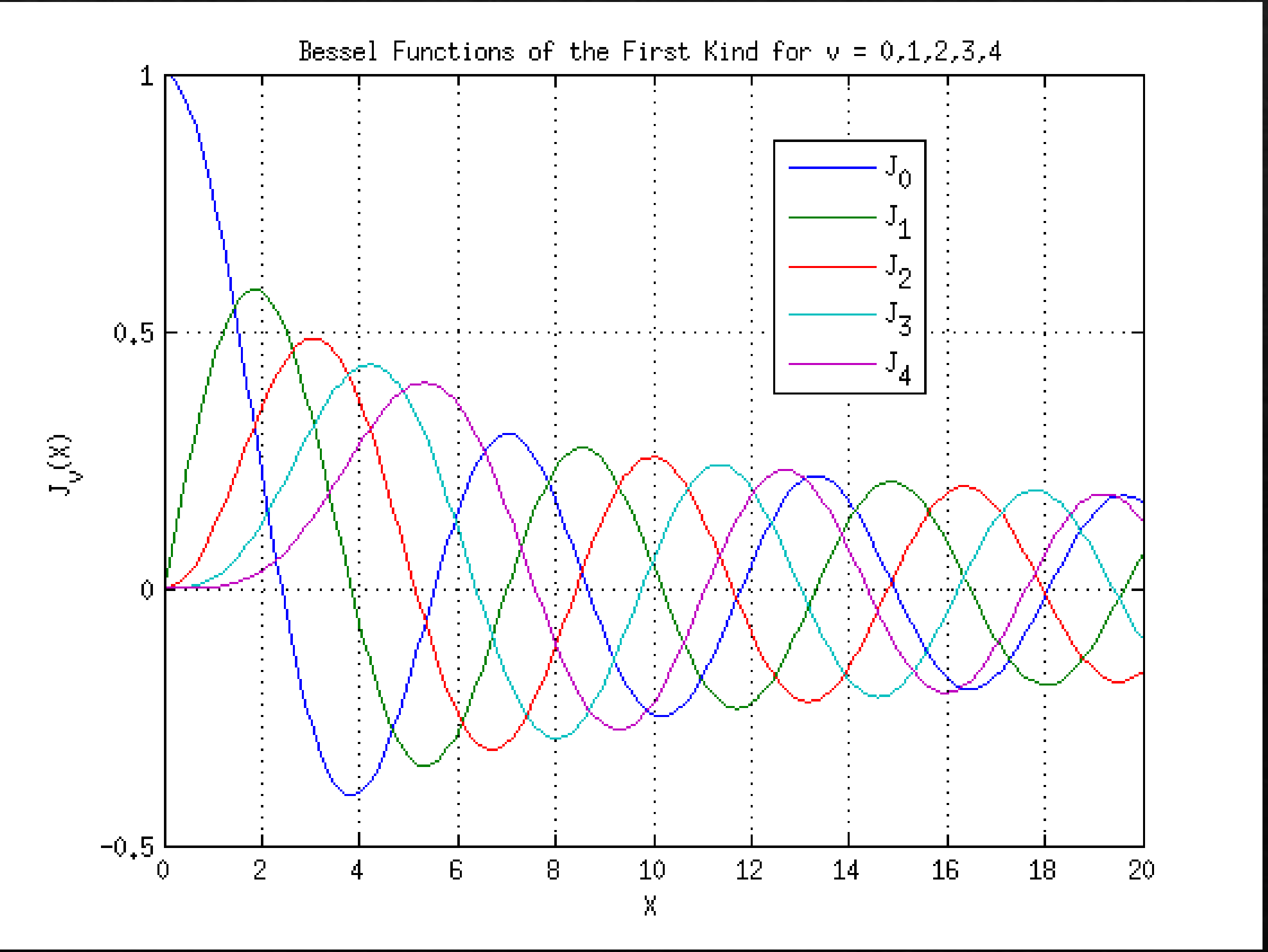
- Solution of the differential equation

$$z^2 \frac{d^2 y}{dz^2} + z \frac{dy}{dz} + \left(z^2 - \nu^2\right) y = 0,$$

- Series expansion

$$J_\nu(z) = \left(\frac{z}{2}\right)^\nu \sum_{(k=0)}^{\infty} \frac{\left(\frac{-z^2}{4}\right)^k}{k!\,\Gamma(\nu + k + 1)}$$

```
X = 0:0.1:20;
J = zeros(5,201);
for i=0:4
    J(i+1,:) = besselj(i,X);
end
plot(X,J,'LineWidth',1.5)
axis([0 20 -.5 1])
grid on;
legend('J_0','J_1','J_2','J_3','J_4','Location','Best')
title('Bessel Functions of the First Kind for v = 0,1,2,3,4')
xlabel('X')
ylabel('J_v(X)')
```



Exercises

- Exercise 1:

  Create an n*n symmetric matrix A in two ways:

  i) use built-in functions up-triangle matrix (triu) and the matrix transpose;

  ii) use control flow (for, if, else, …).

- Exercise 2:

  Use the matrix A from exercise 2 and create a size-n vector b, then solve the linear algebra equation A*x=b with two methods:

  i) calculate the inverse of A.

  ii) calculate LU decomposition of A.

  Compare the computation time and error of the two cases.

Exercises

- Exercise 3:

  Consider the curve parameterized by the equations

  $x(t) = \sin(2t)$, $y(t) = \cos(t)$, $z(t) = t$,

  where $t \in [0, 3\pi]$.

  Create a three-dimensional plot of this curve.

  Compute the arc length of this curve. Tips: The arc length formula says the length of the curve is the integral of the norm of the derivatives of the parameterized equations:

$$\int_0^{3\pi} \sqrt{4\cos(2t)^2 + \sin(t)^2 + 1}\, dt.$$

References

- Matlab document center http://www.mathworks.com/help/
- Matlab: Primer
  http://www.mathworks.com/help/pdf_doc/matlab/getstart.pdf
- Matlab: Programming Fundamentals
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