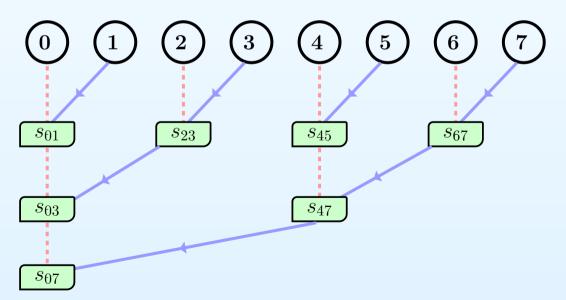


Understanding MPI Applications: A Perspective of Parallel Algorithms

Xiaoxu Guan High Performance Computing, LSU

May 31, 2016





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• Requirements for Parallel Computing







- Requirements for Parallel Computing
- Fundamental Steps of Designing Parallel Algorithms







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- Fundamental Steps of Designing Parallel Algorithms
- Foster's Methodology
 - **Partitioning**;
 - Data Communication;
 - Agglomeration;
 - Mapping;







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- Potential Pitfalls and Maintaining Good Performance







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- Potential Pitfalls and Maintaining Good Performance
- Three MPI Examples
 - Find Prime Numbers
 - MPI Input/Output
 - Matrix-Vector Products
 - Benchmark an MPI Application





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- Fundamental Steps of Designing Parallel Algorithms
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- Further Reading

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Parallel computing



- Requirements for Parallel Computing
- How does MPI meet these requirements?
 - Specify parallel execution single program on multiple data (SPMD) and tasks;
 - Data communication two- and one- side communication (explicit or implicit);
 - Synchronization synchronization functions;
- Data parallelism;





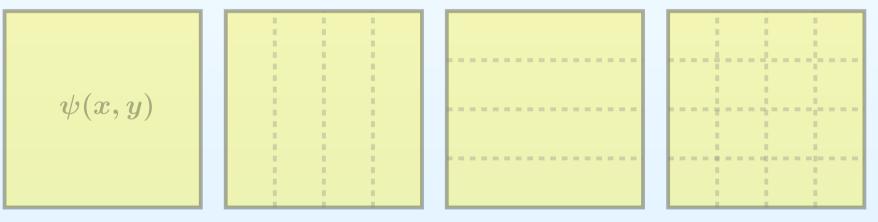
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- Fundamental steps of designing parallel algorithms;
- Shall we design parallel algorithms based on the existing serial algorithms? **Think in parallel!**
- Foster model:
- (1) Partitioning

Divide a large problem into many small ones (tasks);

Domain decomposition;



 Load balance: be sure that each task has the same or similar amount of data to process;







(2) Data communication

- Unless your application doesn't need any exchange of data (trivial parallelism), we have to deal with data communication between different tasks;
- Local communication: for a given task it only needs to talk to a very limited number of other tasks;
- Global communication: a relatively large number of tasks are involved;
- Data communication is not free!
- **Reduce** the number of data communication calls and reduce the amount of data that needs to be transferred;
- Be sure that each MPI task has the same or nearly the same number of communication calls and amount of data;







(3) Agglomeration

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- This is related with the overhead of data communications;
- Trade-off between the number of MPI tasks and the overhead of data communication;
- Combines several small tasks into a larger task;
- Sometimes, reducing the number of MPI tasks might improve the data locality;
- Generally, a rule of thumb is that sending/receiving fewer but longer messages is better than sending/receiving more, but shorter messages;
- More computation and less communication;





- (4) Mapping
- How were multiple tasks assigned to multiple cores?
- Generally, this is probably the most difficult step;
- Maximize CPU utilization and minimize data communication;
- Something beyond load balance: internode and intranode communication ;
- For a given size of the problem and fixed number of cores, how shall we assign tasks to cores: **static** and **dynamic**?
- Static: (1) load balance; (2) regular communication pattern;
 (3) one task/core; (4) each core plays almost the same role;
- Dynamic: **master-worker** model and dispatches tasks to available cores;
- Maintain load balance (computation and communication) and make the code scalable;



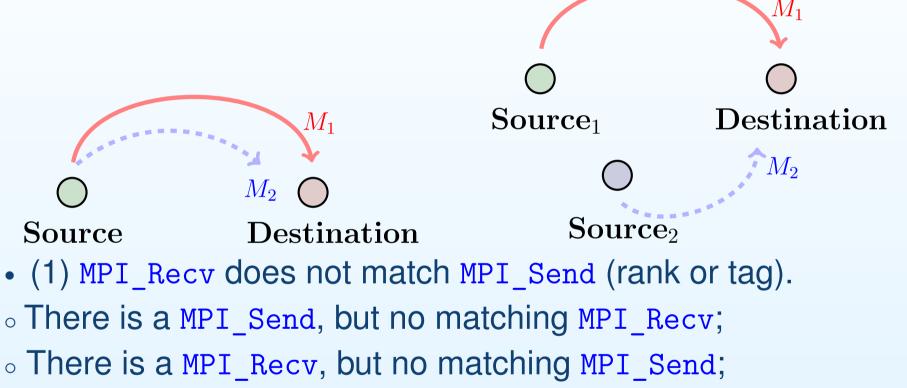


Potential Pitfalls

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- Some common reasons for MPI code hanging or deadlock;
- Message passing should not be overtaking;



 (2) Collective MPI calls are not called so by all MPI ranks in the communicator (say, the issue with only one rank calling MPI_Bcast);





The Sieve of Eratosthenes for Prime Numbers





Find Prime Numbers



- MPI programming for prime number searching below N;
- The serial Sieve of Eratosthenes algorithm;
- One of the ancient but effective iterative methods;
 - Step 1.Generate a list for $2, 3, 4, \cdots$, and N;Step 2.Let k = 2, the first prime in the list;Step 3.Repeat the following procedure:
 - Delete all multiples of k in the region $[k^2, N]$.
 - Locate the smallest number > k. Set the new k to it.
 Until k² > N.

Step 4. All remaining numbers are primes.

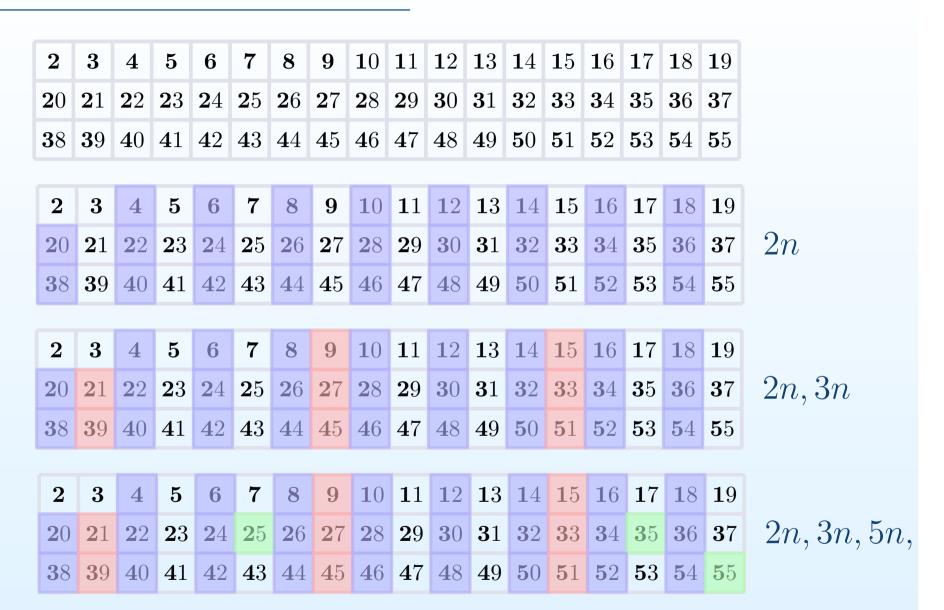
• Let's consider N = 55;



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NFORMATION

The Sieve of Eratosthenes



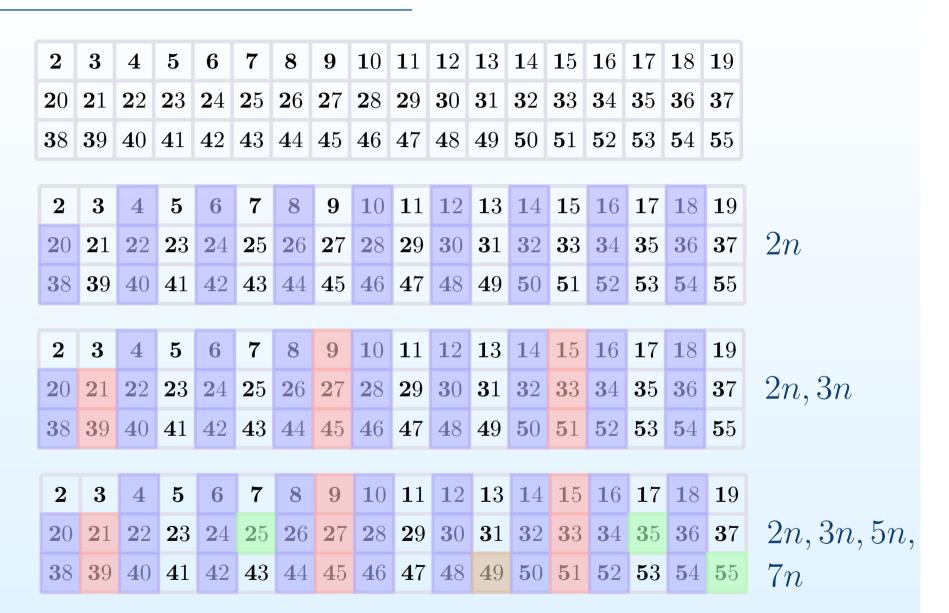




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NFORMATION

The Sieve of Eratosthenes







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- How can we parallelize it using **MPI**?
- Domain (or data) decomposition;
 - Break up the entire list into many smaller consecutive blocks (Partitioning);
 - (2) Shall data communication occur locally or globally (data communication)?
 - (3) We combine the searching multiple of k and marking them out as a larger task (**agglomeration**);
 - (4) We can assign one block to one MPI task (mapping);
- In this case, we have global data communication, because each MPI task needs to know the value of k;
- How often do we need to make data communication?



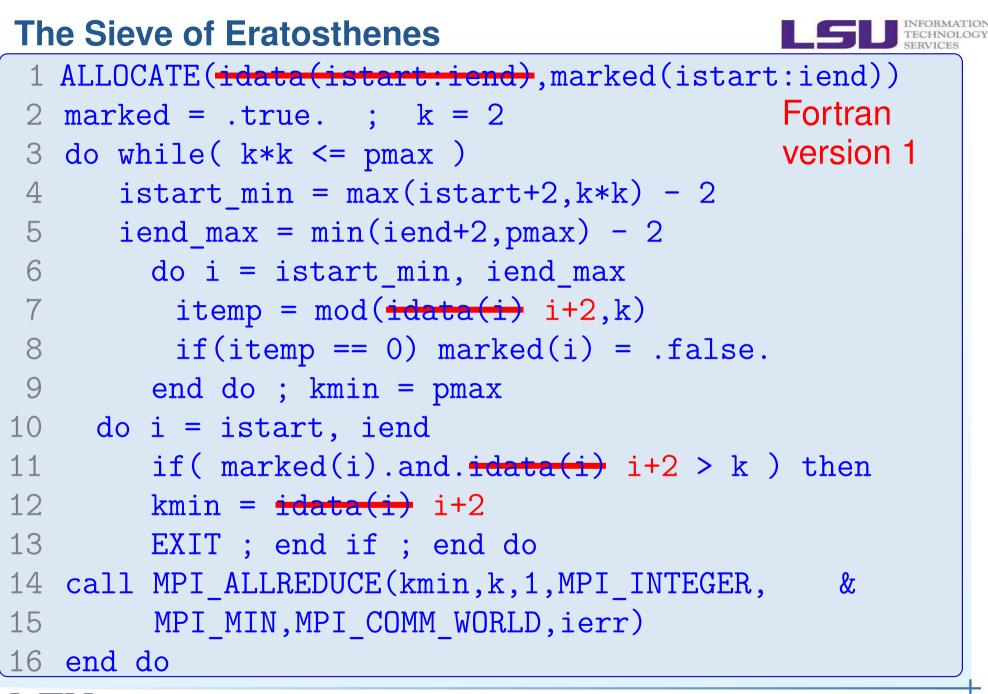
- Load balance: Let's consider N = 2016 on 10 cores; block size is 201 for all cores, except the last task has 206;
- Can we do better?
 - (1) $r = \mod (N-1,p);$
 - (2) block size of $\lceil (N-1)/p \rceil$ for the first of the *r* MPI tasks,
 - (3) block size of $\lfloor (N-1)/p \rfloor$ for the rest of the p-r MPI tasks;
- 5 MPI tasks have the block size of 201, and the rest of the 5 tasks have the block size of 202;
- A much **better** data distribution and it's quite general!
- Version 0: (1) istart, iend; (2) primes(:), marked(:); (3) search all integers in [istart,iend] for multiple of k; (4) call MPI_Allreduce to determine the next global k;





```
1 ALLOCATE(idata(istart:iend),marked(istart:iend))
                                              Fortran
 2 \text{ marked} = .true.; k = 2
                                              version 0
 3 do while( k*k <= pmax )
      istart min = max(istart+2,k*k) - 2
 4
 5
      iend max = min(iend+2,pmax) - 2
 6
        do i = istart min, iend max
 7
          itemp = mod(idata(i),k)
         if(itemp == 0) marked(i) = .false.
 8
 9
        end do ; kmin = pmax
10
    do i = istart, iend
        if( marked(i).and.idata(i) > k ) then
11
12
        kmin = idata(i)
13
        EXIT ; end if ; end do
14 call MPI ALLREDUCE(kmin,k,1,MPI INTEGER,
                                                 X.
        MPI MIN, MPI COMM WORLD, ierr)
15
  end do
16
```

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• Can we do better? Take a look at the do loop (lines 6-9);

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- Can we do even better? Delete all even integers!

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Exercises 1 & 2



• **Exercise 1**: Based on the mpi_primes_v3, replace the collective MPI_Allreduce by other MPI data communications.

[Hint] Break up MPI_Allreduce into two MPI commands.

• Exercise 2: We have found out the number of primes below N. Starting from mpi_primes_v3, add the necessary code segment to print out all the primes from small to large below N.

[Hint] Let all other MPI tasks send the data to the master, and let the master print the primes out.





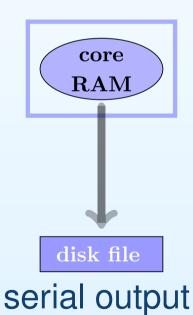








- The next problem we face is the parallel MPI I/O;
- (1) Assign **one** MPI task to take care of all the I/O, and send (receive) the necessary data to (from) other MPI tasks;
- (2) Each MPI task handles the **same** input or output file, but works on a **different** part of the file (the **best** solution);

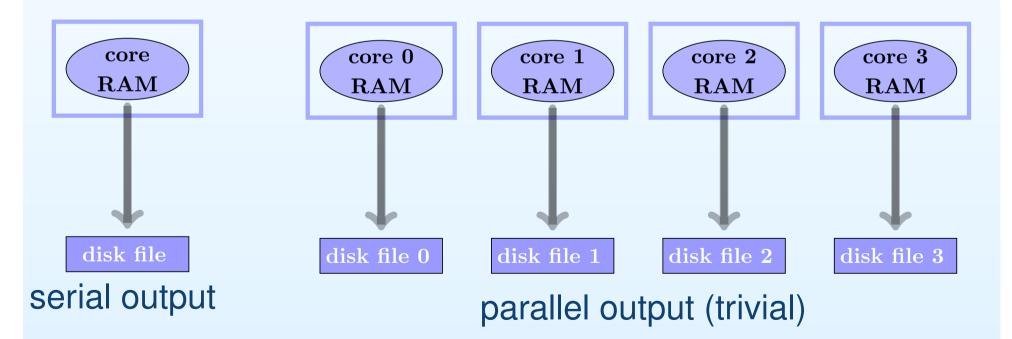


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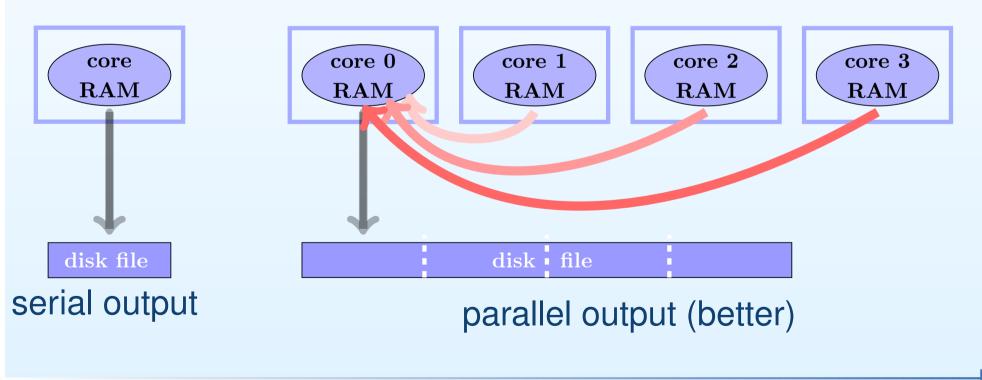




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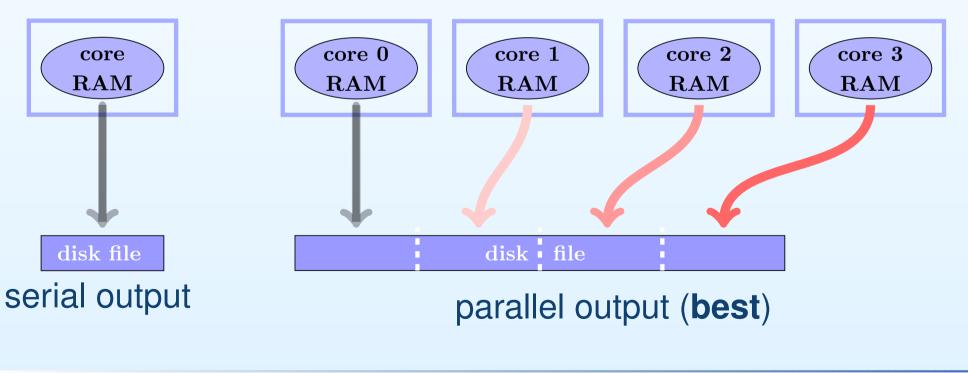
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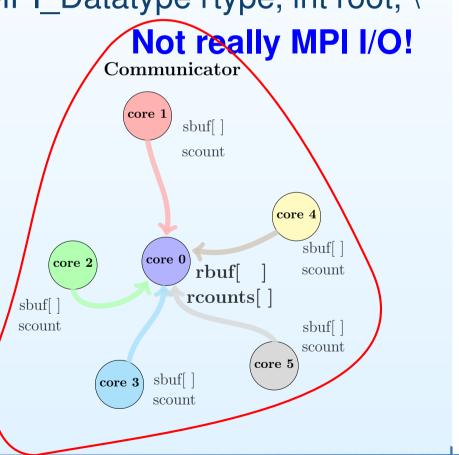


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- (1) One MPI collects all info, and makes the I/O; the amount of data gathered from all MPI tasks may not be the same;
- MPI_Gatherv(*sbuf, int scount, MPI_Datatype stype, \
 *rbuf, int *rcounts, int *displs, MPI_Datatype rtype, int root, \
 MPI_Comm comm);
 Not really MPI I/O!

For root, define

```
displs=(int *) malloc
(numprocs*sizeof(int));
displs[i] = \sum_{k=0}^{i-1} s[k];
displs[0]=0;
displs[1]=s[0];
displs[2]=s[0]+s[1];
displs[3]=s[0]+s[1]+s[2];
```





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• (2) Each MPI task handles the same input or output file;

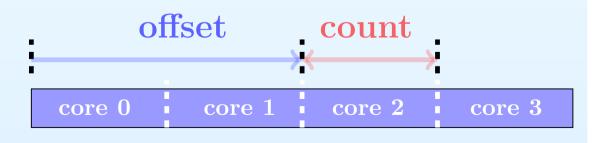
int MPI_File_open(MPI_Comm comm,char *dfilename, \
 int amode, MPI_Info info, MPI_File *fh);

int MPI_File_set_view(MPI_File fh,MPI_Offset disp,\
 MPI_Datatype etype, MPI_Datatype filetype, \
 char *datarep, MPI_Info info);

int MPI_File_write_at(MPI_File fh, MPI_Offset
 offset, *buf, int count, MPI_Datatype datatype,
 MPI_Status *status);

For all MPI tasks, set disp=0; (global offset) local offset; For instance, for core 2:

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• MPI also supports **nonblocking** I/O;

int MPI_File_iwrite_at(MPI_File fh, MPI_Offset offset, \
*buf, int count, MPI_Datatype datatype, \
MPI_Request *request);

int MPI_Wait(MPI_Request *request, MPI_Status *status);

- Overlap computation or communication with the I/O;
- Note that files are in **binary** or **unformatted**;
- How can we assure that the output file makes sense?
- One way to check is to measure the file size and compare with what it should be;

int **MPI_File_get_size**(MPI_File **fh**, MPI_Offset ***size**);

• size is in bytes;







MPI Matrix-Vector Multiplications





MPI matrix-vector products



- Matrix-vector multiplications are very common in physics, applied math, and engineering;
- Many practical problems can be represented in the matrix form, and it is likely matrix-vector products and systems of linear equations need to be handled. Iterative methods in linear algebra depend on matrix-vector products;
- Matrix-matrix products can be reduced to multiple matrix-vector products;
- Let's say we need to compute a **power** of matrices operating on a vector: c = A^k · b = AAA ··· A · b;
- Remember FLOPS for square matrix-matrix product $\sim O(n^3)$, while matrix-vector $\sim O(n^2)$;
- $c_i = \sum_j A_{ij} b_j$

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MPI matrix-vector products

		_												
	c_1		A_{11}	A_{12}	A_{13}	A_{14}	A_{15}	A_{16}	A_{17}	A_{18}	A_{19}	A_{110}		b_1
	c_2													b_2
	c_3													b_3
ĺ	c_4													b_4
ĺ	c_5	_											V	b_5
ľ	c_6	_											×	b_6
ľ	c_7													b_7
	c_8		A_{81}	A_{82}	A_{83}	A_{84}	A_{85}	A_{86}	A_{87}	A_{88}	A ₈₉	A_{810}		b_8
ľ	c_9						1							b_9
	c_{10}													b_{10}
	\boldsymbol{c}	c A												b



- At least three options *A*:
 - (1) Column-wise block decomposition;
 - (2) Row-wise block decomposition;
 - (3) 2D domain decomposition;

In all three cases, how should we distribute vectors b and c?

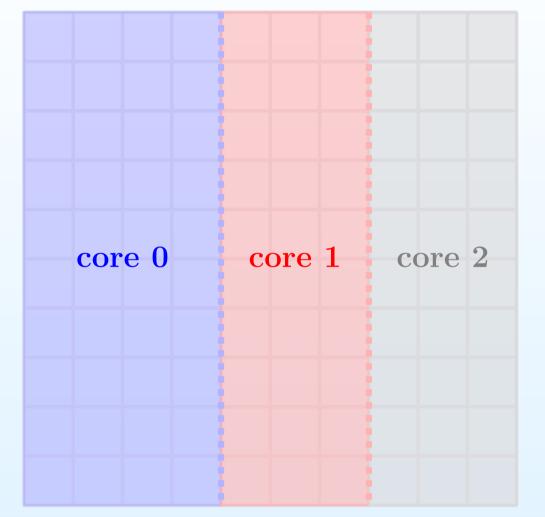




MPI matrix-vector products



• (1) Column-wise block decomposition;

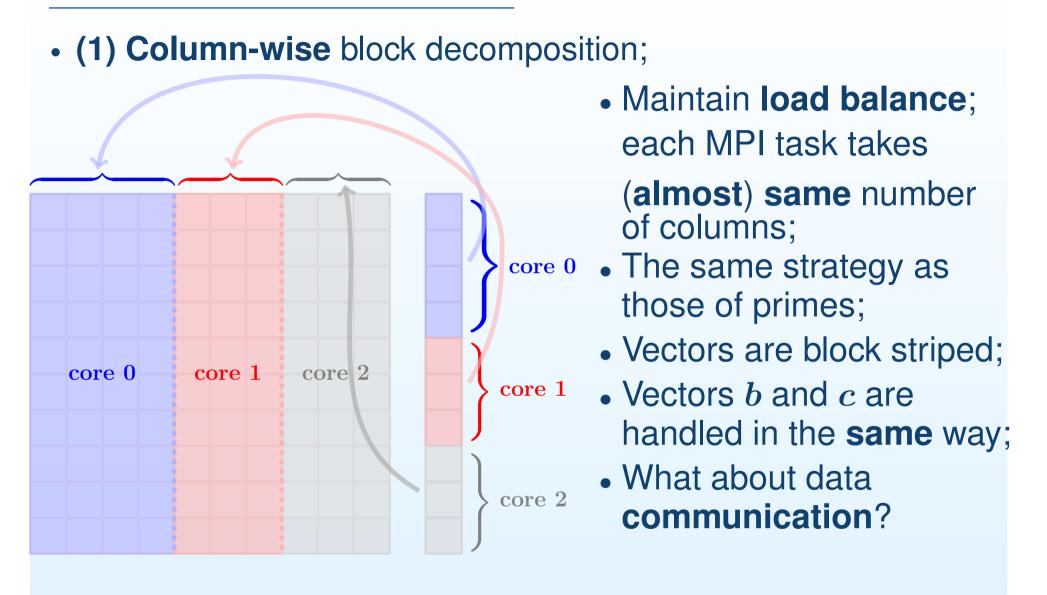


- Maintain load balance; each MPI task takes (almost) same number of columns;
- The same strategy as those of primes;
- Vectors are block striped;
- Vectors b and c are handled in the same way;







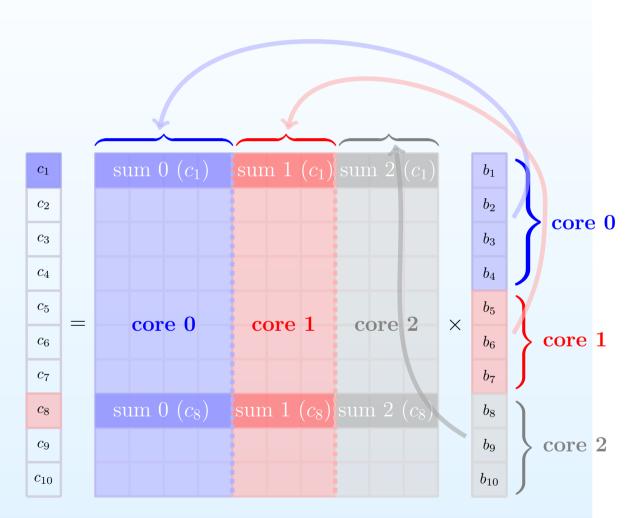






• Data communication in column-wise decomposition;

- **1**. A MPI task computes its own contributions to a vector element;
- **2.** A task needs to gather the contributions from all other tasks;
- **3**. It sums up all contributions;
- **4**. Different tasks may have different numbers of vector elements;
- 5. Use MPI_Alltoallv;









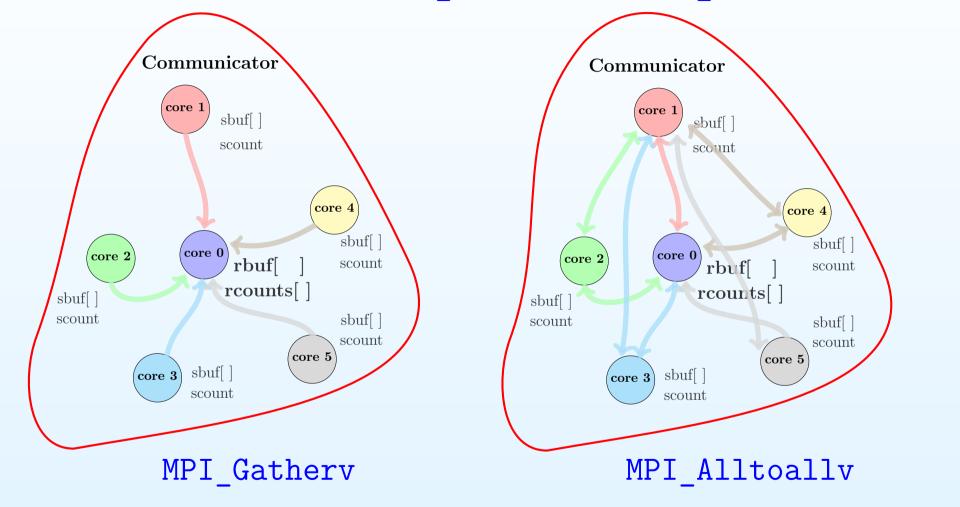
- Data communication in column-wise decomposition;
- We use MPI I/O to read in all matrix and vector elements;

1	do i = 1, nsize	
2	$c_local_temp(i) = 0.0_idp$	Fortran
3	do j = istart, iend	version 0
4	<pre>c_local_temp(i) = c_local_temp(i)</pre>	&
5	+ matrix(i,j) * vector_inp(j)	
6	end do	
7	end do subroutine matveo	c()

- call MPI_Alltoallv;
- The difference between MPI_Gatherv and MPI_Alltoallv;
- After gathering all pieces of data from other tasks and itself, each MPI task needs to reorganize the data to obtain the final output vector;



• The difference between MPI_Gatherv and MPI_Alltoallv;





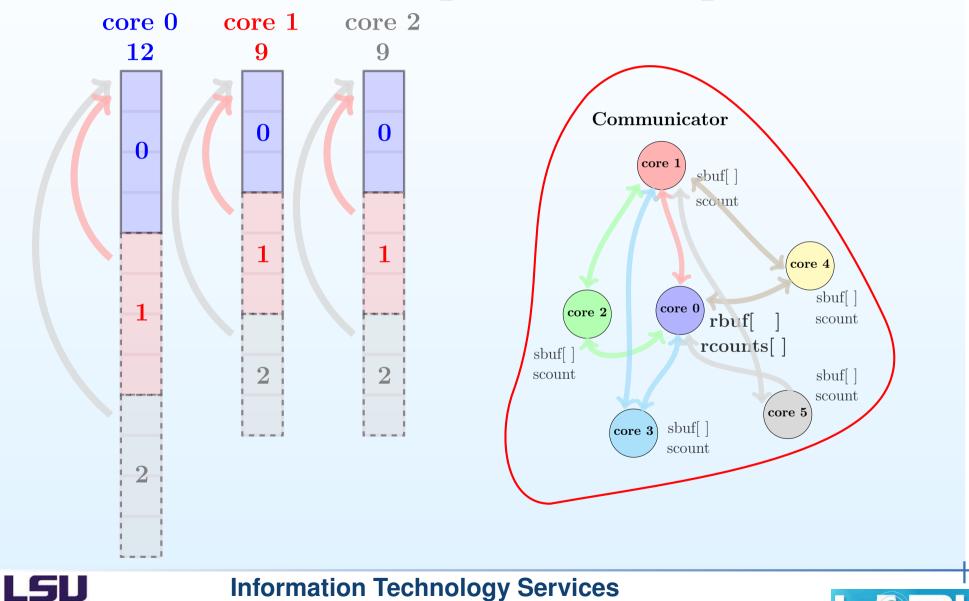


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• The difference between MPI_Gatherv and MPI_Alltoallv;



- Again, can we do **better**?
- Remember Fortran stores 2D arrays in column-wise, while C stores 2D arrays in row-wise;
- Fortran version 0 is not optimized in terms of the way the matrix elements are addressed;

1	<pre>c_local_temp = 0.0_idp</pre>	Fortran
2	do j = istart, iend	version 1
3	do i = 1, nsize	
4	<pre>c_local_temp(i) = c_local_temp(i) &</pre>	
5	+ matrix(i,j) * vector_inp(j)	
6	end do	
7	<pre>end do subroutine matvec()</pre>	

• Exchange the loops;

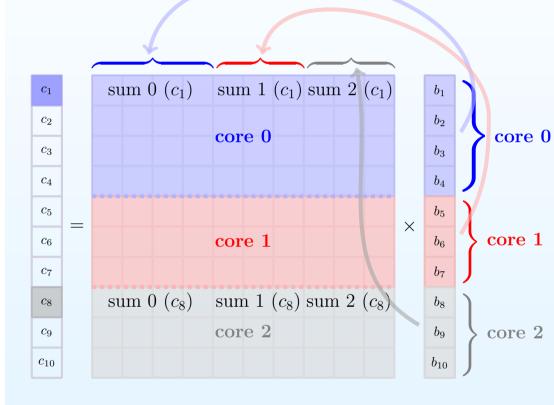
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• The compilers wouldn't do this for you;



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• (2) Row-wise block decomposition;

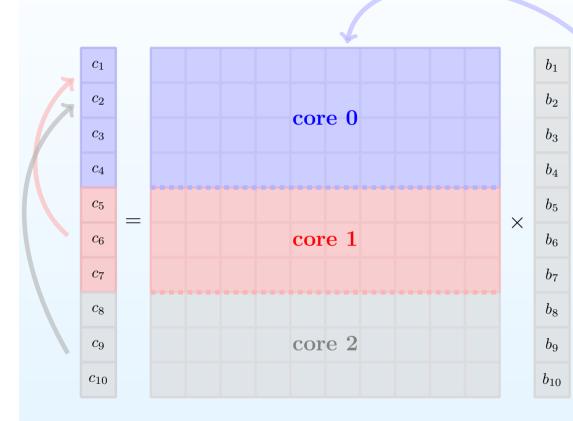


- Maintain load balance; each MPI task takes (almost) same number of rows;
- The same strategy as those of column-wise;
- What about vectors?
- Vectors b and c are handled in the same way;





• (2) Row-wise block decomposition;



- Maintain load balance;
 each MPI task takes
 (almost) same number
 of rows;
- The same strategy as those of column-wise;
- How about vectors?
- Each MPI task has entire vectors b and c;
 - Data communications for vector *c*;





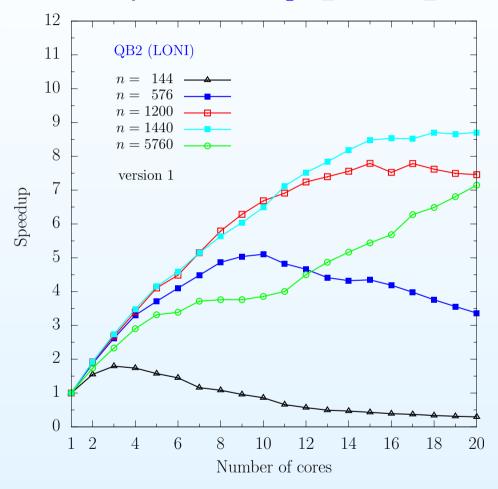


- Increasing FLOPS per unit time is one of our endless goals in the HPC community;
- Maintaining parallel scalability: how an MPI code behave with increasing numbers of cores or threads;
- Before we are able to benchmark an MPI application, be sure that the results are correct!
- Strong scaling and weak scaling from different perspectives of measurements;
- We are interested to spot this information in your allocation proposals!
- Rank 0 measures time_s = MPI_Wtime(); time_e = MPI_Wtime(); elapsed_time = time_e - time_s in second;
- Average wall-clock time or a shortest wall-clock time?





• Examples: run mpi_matve_v1 and _v2 (10^4 times of $c = A \cdot b$);

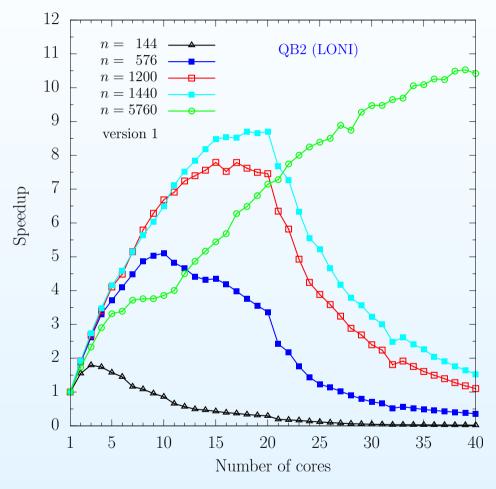


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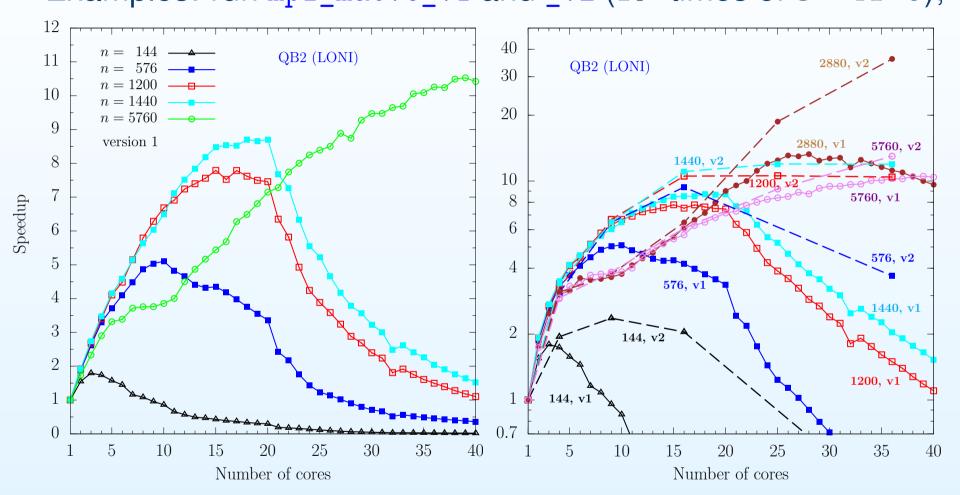
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• Examples: run mpi matve v1 and v2 (10^4 times of $c = A \cdot b$);



Performance really depends on the algorithms, problem

sizes, etc;



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Exercises 3



- Exercise 3: Run mpi_matvec_v1 for matrix sizes of 144, 576, 1200, and 1440, respectively. The number of MPI tasks is from 1 to 16. Benchmark the wall-clock time.
 - (1) What is the max speedup you could get?
 - (2) How would you explain the performance difference for small and large matrices?

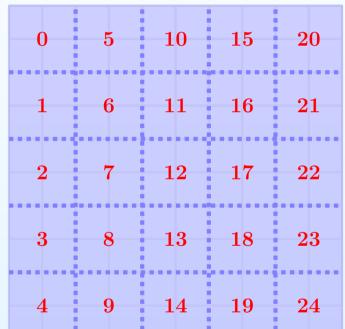






- (3) 2D domain decomposition (DD);
- 1D column- and row-wise decomposition are particular cases of 2D domain decomposition;

core 0	core 2
core 1	core 3



More MPI tasks in 2D cases;

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• Vectors are block striped (same as the column-wise case);



- Generally, 2D DD is much more complicated than 1D cases;
- (1) We only consider a square matrix times a vector;
- (2) Assume the number of MPI tasks is a square number;
- (3) Matrix size should be **dividable** by the number of MPI tasks along row (or column) dimension;
- Create a 2D **Cartesian** (x, y) DD;
- 1 MPI_Cart_create(MPI_COMM_WORLD,2,dimes,bdperiodic,\
 2 topology,&COMM2D);
- 3 MPI_Comm_rank(COMM2D,&my_id);
- 4 MPI_Cart_coords(COMM2D,my_id,2,coords_2d);
- 5 mycoods_x = coords_2d[0];
- 6 mycoods_y = coords_2d[1];

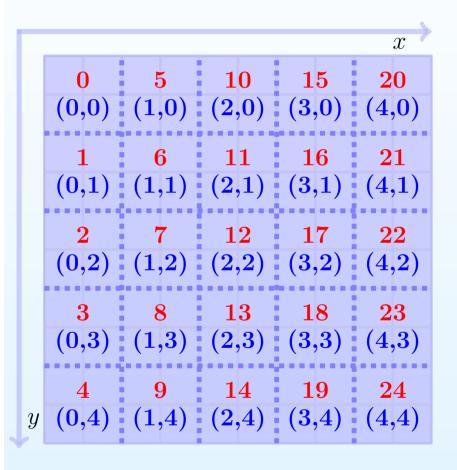
C version 2

- A new communicator COMM2D;
- bdperiodic and topology;





• 2D Cartesian coordinates;



- Need to manipulate matrix elements in row or column patterns;
- Use *x* or *y* coordinates to map operation on MPI tasks;
- Analyze data communication for vectors *b* and *c*;









 $\mathbf{20}$

 \dot{x}

20

(4,0)

21

(4,1)

22

(4,2)

23

(4,3)

 $\mathbf{24}$

15

15

(3,0)

16

17

(3,2)

18

(3,3)

19

(3,4) (4,4)

.

(3,1)

10

10

12

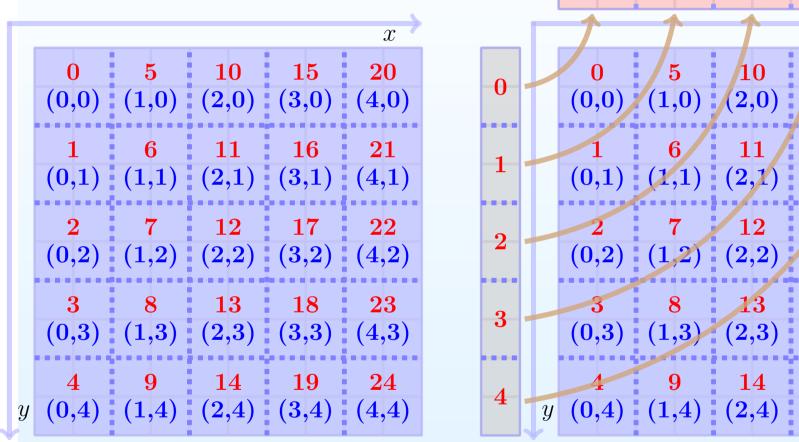
13

14

5

()

2D Cartesian coordinates;



 From the left-most cores to the top-most cores; then roll down for all rows;

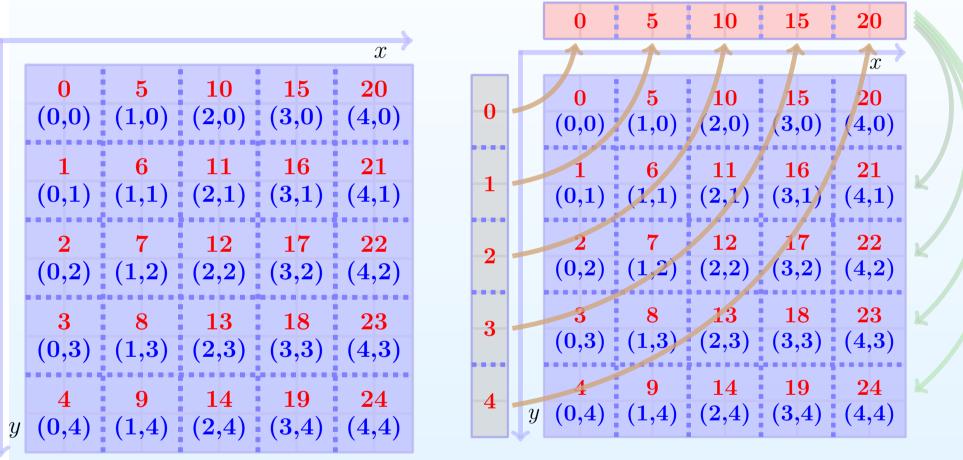


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• 2D Cartesian coordinates;



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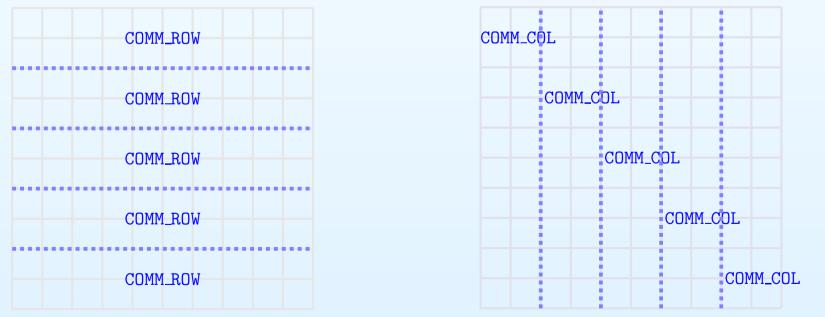
- 4 MPI_Comm_split(MPI_COMM_WORLD,mycoods_x,mycoods_y,\
 5 &COMM_COL);
- 6 MPI_Comm_rank(COMM_COL,&my_id_col);
- How many COMM_ROW or COMM_COL do we have?







- 4 MPI_Comm_split(MPI_COMM_WORLD,mycoods_x,mycoods_y,\
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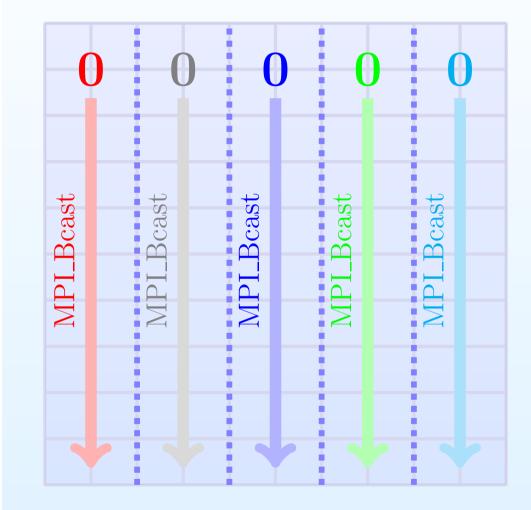
MPI_Bcast(vector,chunk,MPI_DOUBLE,0,COMM_COL);







MPI_Bcast(vector,chunk,MPI_DOUBLE,0,COMM_COL);



- Should we use COMM_ROW or COMM_COL?
- Several small subsets of MPI_WORLS_WORLD;
- In this case, all subsets are named in the same way;
- All cores in the same column have the same chunk of the vector;
- Data communication within the same subset;







- After **local** matrix-vector products, each MPI task has its own contribution to the final vector *c*;
- 1 MPI_Reduce(c_local_temp,vector_out,chunk, \
 2 MPI_DOUBLE,MPI_SUM,0,COMM_ROW);





Exercises 4 & 5



- Exercise 4: In the code mpi_matvec_v2.c (.f90), MPI_Reduce was used, so MPI task with rank 0 gathered the final answer. Can we replace MPI_Reduce with MPI_Allreduce?
- Exercise 5: In the same code, the post data communication was done in the main program and was separated from the function (routine) of matvec. Make the other version (say, mpi_matvec_v3) in such a way that the post data communication is carried in the function of matvec;







Using MPI, Portable Parallel Programming with the Message-Passing Interface, W. Gropp, E. Lusk, and A. Skjellum (The MIT Press, 2014).

Parallel Programming in C with MPI and OpenMP, M. J. Quinn (McGraw Hill, 2004).

Questions?

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